Demo file for case based SEM

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In[1]:=
```
Get["~/Dropbox/Statistik/sem_studie/newSEM2/ncbSEM_V11.wl"];
Get["~/Dropbox/Statistik/sem_studie/newSEM2/RBollen.wl"];
fastM = {"W1", "Wn", "Ww(Wn)"};
slowM = {"W^{2a}", "W^{2a0}"}; (* Methods *)
Nsimu = 100;  (* # simulations *)
(* Bollen's democrarcy model *)
BollenEQ = {{dem60 == b1 * ind60 + u1, e01}, {dem65 == b2 * ind60 + b3 * dem60 + u2, e02},
   {x1 == 1 * ind60 + t1, ee1}, {x2 == c2 * ind60 + t2, ee2}, {x3 == c3 * ind60 + t3, ee3},
   {y1 == 1 * dem60 + s1, e1}, {y2 == d2 * dem60 + s2, e2}, {y3 == d3 * dem60 + s3, e3},
   {y4 == d4 * dem60 + s4, e4}, {y5 == 1 * dem65 + s5, e5}, {y6 == d6 * dem65 + s6, e6},
   {y7 == d7 * dem65 + s7, e7}, {y8 == d8 * dem65 + s8, e8} };
BollenEQ0 = BollenEQ /. { s1 → 0, s2 → 0, s3 → 0, s4 → 0, s5 → 0,
    s6 → 0, s7 → 0, s8 → 0, t1 → 0, t2 → 0, t3 → 0, u1 → 0, u2 → 0 };
Blvars = {ind60, dem60, dem65}; NVR[0] := 0;
Bobserved = {y1, y2, y3, y4, y5, y6, y7, y8, x1, x2, x3};
NVR[sig_] := RandomVariate[NormalDistribution[0, sig], 1][[1]];
NVR[sig_, n_] := RandomVariate[NormalDistribution[0, sig], n];
UVR[sig_, n_] := RandomVariate[UniformDistribution[{-sig, sig} * Sqrt[3]], n];
UVR2[sig_, n_] :=
  sig * 10 / 9 * (3 * RandomVariate[UniformDistribution[{0, 1}], n] ^ 2 - 1);
UVR2i[sig_, n_] :=
  -sig * 10 / 9 * (3 * RandomVariate[UniformDistribution[{0, 1}], n] ^ 2 - 1);
UVR3[sig_, n_] :=
  sig * 4 * (RandomVariate[UniformDistribution[{0.0001, 1}], n] ^ 0.5 - 2 / 3);

Options[SimData] = {simerr → False, latnorm → 0, errnorm → 0, Loffsets → {0, 0, 0}};
SimData[n_, {pb1_, pb2_, pb3_, pc2_, pc3_, pd2_, pd3_, pd4_, pd6_, pd7_, pd8_},
    {sigX1_, sigX2_, sigX3_, sigY1_, sigY2_, sigY3_, sigY4_,
     sigY5_, sigY6_, sigY7_, sigY8_, sig1_, sig2_}, OptionsPattern[]] :=
  Module[{i, IND60, DEM60, k, ksol,
    DEM65, X1, X2, X3, Y1, Y2, Y3, Y4, Y5, Y6, Y7, Y8, res, ee,
    EVR, EVRb, LVR, E01, E02, EE1, EE2, EE3, E1, E2, E3, E4, E5, E6, E7, E8},
   EVR = Switch[OptionValue@errnorm, 0, NVR, 1, UVR, 2, UVR2, 3, UVR2i, 4, UVR3];
   EVRb[sig_, nn_] := -EVR[sig, nn];
   LVR = Switch[OptionValue@latnorm, 0, NVR, 1, UVR, 2, UVR2, 3, UVR2i, 4, UVR3];
   IND60 = LVR[1, n] + OptionValue[Loffsets][[1]];
   X1 = 1.0 * IND60 + EVR[sigX1, n]; EE1 = X1 - IND60;
   X2 = pc2 * IND60 + EVRb[sigX2, n]; EE2 = X2 - pc2 * IND60;
   X3 = pc3 * IND60 + EVRb[sigX3, n]; EE3 = X3 - pc3 * IND60;
   DEM60 = pb1 * IND60 + EVR[sig1, n] + OptionValue[Loffsets][[2]];
   E01 = DEM60 - pb1 * IND60;
   Y1 = 1.0 * DEM60 + EVRb[sigY1, n];  E1 = Y1 - DEM60;
   Y2 = pd2 * DEM60 + EVRb[sigY2, n];  E2 = Y2 - pd2 * DEM60;
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    Y3 = pd3 * DEM60 + EVR[sigY3, n]; E3 = Y3 - pd3 * DEM60;
    Y4 = pd4 * DEM60 + EVR[sigY4, n]; E4 = Y4 - pd4 * DEM60;
    DEM65 = pb2 * IND60 + pb3 * DEM60 + NVR[sig2, n] + OptionValue[Loffsets][[3]];
    E02 = DEM65 - pb2 * IND60 - pb3 * DEM60;
    Y5 = 1.0 * DEM65 + EVRb[sigY5, n];  E5 = Y5 - 1 * DEM65;
    Y6 = pd6 * DEM65 + EVR[sigY6, n]; E6 = Y6 - pd6 * DEM65;
    Y7 = pd7 * DEM65 + EVR[sigY7, n]; E7 = Y7 - pd7 * DEM65;
    Y8 = pd8 * DEM65 + EVRb[sigY8, n]; E8 = Y8 - pd8 * DEM65;
    If[OptionValue@simerr, ee = NVR[0.3, n];
     Y4 = Y4 + ee;
     Y8 = Y8 + ee;
     E4 = E4 + ee;
     E8 = E8 + ee;];
    res = Transpose[{Y1, Y2, Y3, Y4, Y5, Y6, Y7, Y8, X1, X2, X3}];
    {res, Map[Apply[Rule, #] &,
      Join[{{b1, pb1}, {b2, pb2}, {b3, pb3}, {c2, pc2}, {c3, pc3},
        {d2, pd2}, {d3, pd3}, {d4, pd4},  {d6, pd6}, {d7, pd7}, {d8, pd8}},
       Table[{ind60[i], IND60[[i]]}, {i, 1, n}], Table[{dem60[i], DEM60[[i]]},
        {i, 1, n}], Table[{dem65[i], DEM65[[i]]}, {i, 1, n}]]],
     Map[StandardDeviation, {IND60, DEM60, DEM65}],
     {E01, E02, EE1, EE2, EE3, E1, E2, E3, E4, E5, E6, E7, E8}}];
(* {pb1_,pb2_,pb3_,    pc2_,pc3_,     pd2_,pd3_,pd4_,pd6_,pd7_,pd8_} *)
para0 = {1.2, 0.5, 0.8,   0.7, 0.9,   0.3, 0.9, 1.7,   0.6, 0.4, 1.3};
(* {sigX1_,sigX2_,sigX3_, sigY1_,sigY2_,sigY3_,
 sigY4_,sigY5_,sigY6_,sigY7_,sigY8_,sig1_,sig2_}*)
Spara0 = {0.1, 0.2, 0.3,   0.2, 0.1, 0.2, 0.3,  0.2, 0.1, 0.2, 0.3,   0.3, 0.2};
Spara0b = {0.2, 0.3, 0.1,   0.2, 0.1, 0.2, 0.3,  0.2, 0.1, 0.2, 0.2,   0.4, 0.3};
Spara0c = {0.2, 0.2, 0.1,   0.2, 0.1, 0.2, 0.3, 0.2, 0.3, 0.2, 0.2,   0.5, 0.3};
Spararules = Map[Apply[Rule, #] &, Transpose[{{sigX1, sigX2, sigX3, sigY1, sigY2,
     sigY3, sigY4, sigY5, sigY6, sigY7, sigY8, sig1, sig2}, Spara0}]];
BestWeights = {e01 → 1 / sig1^2,  e02 → 1 / sig2^2,
    ee1 → 1 / sigX1^2, ee2 → 1 / sigX2^2, ee3 → 1 / sigX3^2,
    e1 → 1 / sigY1^2, e2 → 1 / sigY2^2, e3 → 1 / sigY3^2, e4 → 1 / sigY4^2,
    e5 → 1 / sigY5^2, e6 → 1 / sigY6^2,
    e7 → 1 / sigY7^2, e8 → 1 / sigY8^2} /. Spararules;
BestWeightsW = Map[Apply[Rule, {weight[#[[1]]], #[[2]]}] &, BestWeights];
wopt = Map[Last, BestWeights] / Apply[Plus, Map[Last, BestWeights]];
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In[•]:=
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(**************** some in depth anaylsis on Bollen's model *****)
n = 500; datt = SimData[n, para0, Spara0, simerr → False, latnorm → 0, errnorm → 0];
val0 = Join[Table[{ind60[i], First[datt][[i, 9]]}, {i, 1, n}],
  Table[{dem60[i], First[datt][[i, 1]]}, {i, 1, n}],
  Table[{dem65[i], First[datt][[i, 5]]}, {i, 1, n}]
 ]; dat = First@datt;
resW2a = slowSEM["W^{2a}", dat, Bobserved, Blvars, BollenEQ, InitValues → val0];
resW2a0 = slowSEM["W^{2a0}", dat, Bobserved, Blvars, BollenEQ, InitValues → val0];
Print[
  {"W2a", {b1, b2, b3} /. resW2a[["sol"]], "W2a0", {b1, b2, b3} /. resW2a0[["sol"]]}];
Print[{MatrixForm[Round[resW2a["EEcors"], 0.01]],
  MatrixForm[Round[resW2a0["EEcors"], 0.01]]}];
Print[{MatrixForm[Round[resW2a["DEcors"], 0.01]],
  MatrixForm[Round[resW2a0["DEcors"], 0.01]]}];
Print[{MatrixForm[Round[resW2a["LEcors"], 0.01]],
  MatrixForm[Round[resW2a0["LEcors"], 0.01]]}];
Print[{"W2a", resW2a@"GOFMC",
   resW2a@"Fmin", "W2a0", resW2a0@"GOFMC", resW2a0@"Fmin"}];
```

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{W2a, {1.25253, 0.439185, 0.858136}, W2a0, {1.19, 0.349508, 0.924668}}
```

$$\left\{\begin{pmatrix} 1. & -0.01 & 0. & 0.03 & 0.02 & 0.05 & -0.07 & -0.02 & -0.01 & -0.01 & 0.02 & -0.07 & 0.01 \\ -0.01 & 1. & 0.03 & 0. & -0.08 & 0.02 & 0.04 & 0. & -0.02 & -0.08 & 0.04 & -0.01 & 0.01 \\ 0. & 0.03 & 1. & -0.01 & 0.02 & 0. & 0.02 & -0.06 & 0.01 & -0.01 & 0.02 & 0.04 & 0. \\ 0.03 & 0. & -0.01 & 1. & 0.04 & -0.02 & -0.03 & 0.1 & -0.03 & 0.06 & -0.05 & -0.04 & 0.02 \\ 0.02 & -0.08 & 0.02 & 0.04 & 1. & -0.06 & 0.02 & 0.1 & 0.03 & 0.02 & -0.07 & 0.01 & -0.03 \\ 0.05 & 0.02 & 0. & -0.02 & -0.06 & 1. & -0.05 & 0.05 & 0.01 & 0.01 & -0.01 & 0.01 & 0.03 \\ -0.07 & 0.04 & 0.02 & -0.03 & 0.02 & -0.05 & 1. & -0.04 & 0.03 & 0.03 & -0.01 & -0.03 & 0.05 \\ -0.02 & 0. & -0.06 & 0.1 & 0.1 & 0.05 & -0.04 & 1. & 0. & 0.03 & -0.02 & -0.04 & 0. \\ -0.01 & -0.02 & 0.01 & -0.03 & 0.03 & 0.01 & 0.03 & 0. & 1. & 0.01 & 0.02 & 0.01 & -0.06 \\ -0.01 & -0.08 & -0.01 & 0.06 & 0.02 & 0.01 & 0.03 & 0.03 & 0.01 & 1. & -0.01 & -0.05 & -0.04 \\ 0.02 & 0.04 & 0.02 & -0.05 & -0.07 & -0.01 & -0.01 & -0.02 & 0.02 & -0.01 & 1. & 0.04 & 0.06 \\ -0.07 & -0.01 & 0.04 & -0.04 & 0.01 & 0.01 & -0.03 & -0.04 & 0.01 & -0.05 & 0.04 & 1. & 0.02 \\ 0.01 & 0.01 & 0. & 0.02 & -0.03 & 0.03 & 0.05 & 0. & -0.06 & -0.04 & 0.06 & 0.02 & 1. \end{pmatrix}\right\},$$

$$\left\{\begin{pmatrix} 0.1 & 0.08 & 0.09 & 0.1 & 0.02 & 0.03 & 0. & 0.03 & -0.15 & -0.14 & -0.14 \\ -0.04 & -0.03 & -0.05 & -0.05 & 0.07 & 0.09 & 0.08 & 0.08 & -0.04 & -0.04 & -0.07 \\ -0.01 & 0. & -0.02 & -0.01 & -0.01 & 0. & 0.01 & -0.01 & 0.09 & -0.01 & 0. \\ -0.02 & -0.02 & 0. & -0.02 & -0.01 & -0.02 & -0.03 & -0.02 & -0.03 & 0.25 & -0.01 \\ -0.03 & -0.01 & 0. & -0.01 & -0.03 & -0.04 & -0.03 & -0.03 & -0.02 & -0.01 & 0.29 \\ 0.14 & -0.02 & 0. & -0.01 & -0.01 & -0.01 & -0.01 & 0. & -0.02 & -0.02 & -0.04 \\ -0.03 & 0.24 & -0.03 & -0.02 & -0.01 & -0.02 & -0.02 & -0.01 & -0.01 & -0.02 & 0. \\ -0.01 & -0.03 & 0.16 & -0.02 & -0.01 & -0.02 & -0.03 & -0.02 & -0.02 & 0.02 & 0.02 \\ -0.01 & 0. & -0.01 & 0.14 & -0.01 & -0.01 & -0.01 & -0.02 & -0.01 & -0.02 & 0. \\ -0.01 & 0. & 0. & -0.01 & 0.12 & -0.02 & -0.03 & -0.02 & -0.01 & 0.01 & 0. \\ -0.01 & -0.01 & -0.01 & -0.01 & -0.01 & 0.11 & 0.01 & 0. & -0.01 & -0.03 & -0.03 \\ -0.03 & -0.03 & -0.03 & -0.03 & -0.03 & -0.02 & 0.29 & -0.02 & -0.01 & -0.02 & -0.01 \\ -0.01 & 0. & -0.01 & -0.02 & -0.02 & 0. & 0. & 0.14 & -0.01 & -0.01 & -0.02 \end{pmatrix}\right\},$$

$$\left\{\begin{pmatrix} 0.26 & 0.22 & 0.24 & 0.25 & 0.17 & 0.17 & 0.14 & 0.17 & -0.03 & -0.02 & -0.02 \\ -0.03 & -0.02 & -0.04 & -0.04 & 0.12 & 0.14 & 0.13 & 0.14 & -0.01 & -0.01 & -0.04 \\ -0.13 & -0.11 & -0.15 & -0.13 & -0.12 & -0.11 & -0.09 & -0.12 & 0.05 & -0.15 & -0.13 \\ -0.02 & -0.02 & 0. & -0.02 & -0.01 & -0.02 & -0.03 & -0.01 & -0.03 & 0.26 & -0.01 \\ -0.03 & -0.01 & 0. & -0.01 & -0.03 & -0.04 & -0.02 & -0.03 & -0.02 & -0.01 & 0.3 \\ 0.26 & 0.07 & 0.1 & 0.09 & 0.09 & 0.08 & 0.08 & 0.09 & 0.07 & 0.06 & 0.05 \\ -0.03 & 0.26 & -0.03 & -0.02 & -0.01 & -0.01 & -0.02 & -0.01 & 0. & -0.01 & 0. \\ -0.03 & -0.05 & 0.16 & -0.04 & -0.03 & -0.04 & -0.05 & -0.04 & -0.04 & 0. & 0. \\ -0.04 & -0.04 & -0.05 & 0.13 & -0.05 & -0.05 & -0.04 & -0.06 & -0.04 & -0.05 & -0.03 \\ 0.07 & 0.07 & 0.07 & 0.07 & 0.21 & 0.06 & 0.04 & 0.05 & 0.07 & 0.09 & 0.08 \\ -0.05 & -0.05 & -0.05 & -0.05 & -0.05 & 0.1 & -0.03 & -0.04 & -0.05 & -0.06 & -0.07 \\ -0.02 & -0.03 & -0.03 & -0.02 & -0.02 & -0.01 & 0.31 & -0.01 & 0. & -0.01 & 0. \\ -0.03 & -0.02 & -0.04 & -0.05 & -0.04 & -0.03 & -0.03 & 0.13 & -0.03 & -0.03 & -0.04 \end{pmatrix}\right\},$$

$$\left\{\begin{pmatrix} -0.15 & 0.1 & 0.02 \\ -0.04 & -0.05 & 0.08 \\ -0.01 & -0.01 & -0.01 \\ -0.02 & -0.02 & -0.02 \\ -0.02 & -0.02 & -0.03 \\ -0.02 & -0.01 & -0.01 \\ -0.01 & -0.03 & -0.02 \\ -0.01 & -0.02 & -0.02 \\ -0.01 & -0.01 & -0.01 \\ 0. & -0.01 & -0.02 \\ -0.01 & -0.01 & 0. \\ -0.01 & -0.03 & -0.02 \\ -0.01 & -0.01 & -0.01 \end{pmatrix}, \begin{pmatrix} 0. & 0.23 & 0.17 \\ 0. & 0. & 0.1 \\ 0. & -0.12 & -0.11 \\ 0. & -0.02 & -0.02 \\ 0. & -0.02 & -0.03 \\ 0.07 & 0.13 & 0.09 \\ -0.01 & 0. & -0.01 \\ -0.03 & 0. & -0.03 \\ -0.04 & 0. & -0.04 \\ 0.08 & 0.08 & 0.09 \\ -0.05 & -0.04 & 0. \\ 0. & -0.01 & 0. \\ -0.03 & -0.03 & 0. \end{pmatrix}\right\}$$

```
{W2a, {0.135095, 0.27422, 0.0831422, 0.0479222},
 0.6428, W2a0, {0.232844, 0.452472, 0.140816, 0.105245}, 0.512192}
```

In[ ]:=
```
dat = First@datt;
resW2a = slowSEM["W^{2a}", dat, Bobserved,
  Blvars, BollenEQ, InitValues → val0, LatentMean0 → True];
resW2a0 = slowSEM["W^{2a0}", dat, Bobserved,
   Blvars, BollenEQ, InitValues → val0, LatentMean0 → True];
Print[
  {"W2a", {b1, b2, b3} /. resW2a[["sol"]], "W2a0", {b1, b2, b3} /. resW2a0[["sol"]]}];
Print[{MatrixForm[Round[resW2a["EEcors"], 0.01]],
  MatrixForm[Round[resW2a0["EEcors"], 0.01]]}];
Print[{MatrixForm[Round[resW2a["DEcors"], 0.01]],
  MatrixForm[Round[resW2a0["DEcors"], 0.01]]}];
Print[{MatrixForm[Round[resW2a["LEcors"], 0.01]],
  MatrixForm[Round[resW2a0["LEcors"], 0.01]]}];
Print[{"W2a", resW2a@"GOFMC",
   resW2a@"Fmin", "W2a0", resW2a0@"GOFMC", resW2a0@"Fmin"}];
```

```
{W2a, {1.25253, 0.439185, 0.858136}, W2a0, {1.19, 0.349508, 0.924668}}
```

$$\left\{\begin{pmatrix} 1. & -0.01 & 0. & 0.03 & 0.02 & 0.05 & -0.07 & -0.02 & -0.01 & -0.01 & 0.02 & -0.07 & 0.01 \\ -0.01 & 1. & 0.03 & 0. & -0.08 & 0.02 & 0.04 & 0. & -0.02 & -0.08 & 0.04 & -0.01 & 0.01 \\ 0. & 0.03 & 1. & -0.01 & 0.02 & 0. & 0.02 & -0.06 & 0.01 & -0.01 & 0.02 & 0.04 & 0. \\ 0.03 & 0. & -0.01 & 1. & 0.04 & -0.02 & -0.03 & 0.1 & -0.03 & 0.06 & -0.05 & -0.04 & 0.02 \\ 0.02 & -0.08 & 0.02 & 0.04 & 1. & -0.06 & 0.02 & 0.1 & 0.03 & 0.02 & -0.07 & 0.01 & -0.03 \\ 0.05 & 0.02 & 0. & -0.02 & -0.06 & 1. & -0.05 & 0.05 & 0.01 & 0.01 & -0.01 & 0.01 & 0.03 \\ -0.07 & 0.04 & 0.02 & -0.03 & 0.02 & -0.05 & 1. & -0.04 & 0.03 & 0.03 & -0.01 & -0.03 & 0.05 \\ -0.02 & 0. & -0.06 & 0.1 & 0.1 & 0.05 & -0.04 & 1. & 0. & 0.03 & -0.02 & -0.04 & 0. \\ -0.01 & -0.02 & 0.01 & -0.03 & 0.03 & 0.01 & 0.03 & 0. & 1. & 0.01 & 0.02 & 0.01 & -0.06 \\ -0.01 & -0.08 & -0.01 & 0.06 & 0.02 & 0.01 & 0.03 & 0.03 & 0.01 & 1. & -0.01 & -0.05 & -0.04 \\ 0.02 & 0.04 & 0.02 & -0.05 & -0.07 & -0.01 & -0.01 & -0.02 & 0.02 & -0.01 & 1. & 0.04 & 0.06 \\ -0.07 & -0.01 & 0.04 & -0.04 & 0.01 & 0.01 & -0.03 & -0.04 & 0.01 & -0.05 & 0.04 & 1. & 0.02 \\ 0.01 & 0.01 & 0. & 0.02 & -0.03 & 0.03 & 0.05 & 0. & -0.06 & -0.04 & 0.06 & 0.02 & 1. \end{pmatrix}\right\},$$

$$\left\{\begin{pmatrix} 0.1 & 0.08 & 0.09 & 0.1 & 0.02 & 0.03 & 0. & 0.03 & -0.15 & -0.14 & -0.14 \\ -0.04 & -0.03 & -0.05 & -0.05 & 0.07 & 0.09 & 0.08 & 0.08 & -0.04 & -0.04 & -0.07 \\ -0.01 & 0. & -0.02 & -0.01 & -0.01 & 0. & 0.01 & -0.01 & 0.09 & -0.01 & 0. \\ -0.02 & -0.02 & 0. & -0.02 & -0.01 & -0.02 & -0.03 & -0.02 & -0.03 & 0.25 & -0.01 \\ -0.03 & -0.01 & 0. & -0.01 & -0.03 & -0.04 & -0.03 & -0.03 & -0.02 & -0.01 & 0.29 \\ 0.14 & -0.02 & 0. & -0.01 & -0.01 & -0.01 & -0.01 & 0. & -0.02 & -0.02 & -0.04 \\ -0.03 & 0.24 & -0.03 & -0.02 & -0.01 & -0.02 & -0.02 & -0.01 & -0.01 & -0.02 & 0. \\ -0.01 & -0.03 & 0.16 & -0.02 & -0.01 & -0.02 & -0.03 & -0.02 & -0.02 & 0.02 & 0.02 \\ -0.01 & 0. & -0.01 & 0.14 & -0.01 & -0.01 & -0.01 & -0.02 & -0.01 & -0.02 & 0. \\ -0.01 & 0. & 0. & -0.01 & 0.12 & -0.02 & -0.03 & -0.02 & -0.01 & 0.01 & 0. \\ -0.01 & -0.01 & -0.01 & -0.01 & -0.01 & 0.11 & 0.01 & 0. & -0.01 & -0.03 & -0.03 \\ -0.03 & -0.03 & -0.03 & -0.03 & -0.03 & -0.02 & 0.29 & -0.02 & -0.01 & -0.02 & -0.01 \\ -0.01 & 0. & -0.01 & -0.02 & -0.02 & 0. & 0. & 0.14 & -0.01 & -0.01 & -0.02 \end{pmatrix}\right\},$$

$$\left\{\begin{pmatrix} 0.26 & 0.22 & 0.24 & 0.25 & 0.17 & 0.17 & 0.14 & 0.17 & -0.03 & -0.02 & -0.02 \\ -0.03 & -0.02 & -0.04 & -0.04 & 0.12 & 0.14 & 0.13 & 0.14 & -0.01 & -0.01 & -0.04 \\ -0.13 & -0.11 & -0.15 & -0.13 & -0.12 & -0.11 & -0.09 & -0.12 & 0.05 & -0.15 & -0.13 \\ -0.02 & -0.02 & 0. & -0.02 & -0.01 & -0.02 & -0.03 & -0.01 & -0.03 & 0.26 & -0.01 \\ -0.03 & -0.01 & 0. & -0.01 & -0.03 & -0.04 & -0.02 & -0.03 & -0.02 & -0.01 & 0.3 \\ 0.26 & 0.07 & 0.1 & 0.09 & 0.09 & 0.08 & 0.08 & 0.09 & 0.07 & 0.06 & 0.05 \\ -0.03 & 0.26 & -0.03 & -0.02 & -0.01 & -0.01 & -0.02 & -0.01 & 0. & -0.01 & 0. \\ -0.03 & -0.05 & 0.16 & -0.04 & -0.03 & -0.04 & -0.05 & -0.04 & -0.04 & 0. & 0. \\ -0.04 & -0.04 & -0.05 & 0.13 & -0.05 & -0.05 & -0.04 & -0.06 & -0.04 & -0.05 & -0.03 \\ 0.07 & 0.07 & 0.07 & 0.07 & 0.21 & 0.06 & 0.04 & 0.05 & 0.07 & 0.09 & 0.08 \\ -0.05 & -0.05 & -0.05 & -0.05 & -0.05 & 0.1 & -0.03 & -0.04 & -0.05 & -0.06 & -0.07 \\ -0.02 & -0.03 & -0.03 & -0.02 & -0.02 & -0.01 & 0.31 & -0.01 & 0. & -0.01 & 0. \\ -0.03 & -0.02 & -0.04 & -0.05 & -0.04 & -0.03 & -0.03 & 0.13 & -0.03 & -0.03 & -0.04 \end{pmatrix}\right\},$$

$$\left\{\begin{pmatrix} -0.15 & 0.1 & 0.02 \\ -0.04 & -0.05 & 0.08 \\ -0.01 & -0.01 & -0.01 \\ -0.02 & -0.02 & -0.02 \\ -0.02 & -0.02 & -0.03 \\ -0.02 & -0.01 & -0.01 \\ -0.01 & -0.03 & -0.02 \\ -0.01 & -0.02 & -0.02 \\ -0.01 & -0.01 & -0.01 \\ 0. & -0.01 & -0.02 \\ -0.01 & -0.01 & 0. \\ -0.01 & -0.03 & -0.02 \\ -0.01 & -0.01 & -0.01 \end{pmatrix}, \begin{pmatrix} 0. & 0.23 & 0.17 \\ 0. & 0. & 0.1 \\ 0. & -0.12 & -0.11 \\ 0. & -0.02 & -0.02 \\ 0. & -0.02 & -0.03 \\ 0.07 & 0.13 & 0.09 \\ -0.01 & 0. & -0.01 \\ -0.03 & 0. & -0.03 \\ -0.04 & 0. & -0.04 \\ 0.08 & 0.08 & 0.09 \\ -0.05 & -0.04 & 0. \\ 0. & -0.01 & 0. \\ -0.03 & -0.03 & 0. \end{pmatrix}\right\}$$

```
{W2a, {0.135095, 0.27422, 0.0831422, 0.0479222},
 0.6428, W2a0, {0.232844, 0.452472, 0.140816, 0.105245}, 0.512192}
```

In[ ]:=
```
Length@dat
```
Out[ ]=
```
500
```

In[ ]:=
```
(********************************** Bollen's democracy
 model ******************************)t0 = AbsoluteTime[];
ST = SemTest[Nsimu, {300 },
  (First@SimData[#, para0, Spara0, simerr → False, latnorm → 0, errnorm → 0]) &,
  fastM, slowM, {b1, b2, b3}, {1.2, 0.5, 0.8},
  {Bobserved, Blvars, BollenEQ}, LatInit → {9, 1, 5}, bollen → True];
Print@ST["ltx"]; Print[AbsoluteTime[] - t0]
```
```
------------------------------------------------------ 300
```

Method	{b1, b2, b3}	std	#fails	avg time	avg GOF	SD GOF
lavaan	{1.197, 0.504, 0.796}	{0.022, 0.067, 0.054}	0	0.6	{}	{}
W1	{1.195, 0.902, 0.475}	{0.023, 0.226, 0.181}	0	61.	0.243	0.006
Wn	{0.989, 0.428, 0.673}	{0.42, 0.162, 0.273}	0	9.4	0.374	0.283
Ww(Wn)	{1.185, 0.237, 1.003}	{0.035, 5.576, 4.428}	0	19.2	0.247	0.009
W^{2a}	{1.243, 0.5, 0.807}	{0.022, 0.071, 0.056}	4	167.6	0.137	0.003
W^{2a0}	{1.173, 0.455, 0.847}	{0.055, 0.21, 0.188}	1	226.6	0.239	0.022

```
\begin{table}
         \caption{Titel (100 simulations for each $n$) }
        {\footnotesize\begin{tabular}{cccccccc}\hline
$n$ & var   & ML  & $W_1$  & $W_n$  & $Ww(W_n)$  & $W^{2a}$  & $W^{2a0}$ \\
300 & $b_1$ & 0. & 0. & -0.21 & -0.01 & 0.04 & -0.03\\
 & 1.2 & (0.02) & (0.02) & (0.42) & (0.04) & (0.02) & (0.05)\\
 & RMSE & 0.02 & 0.02 & 0.47 & 0.04 & 0.05 & 0.06\\
 & $b_2$ & 0. & 0.4 & -0.07 & -0.26 & 0. & -0.05\\
 & 0.5 & (0.07) & (0.23) & (0.16) & (5.58) & (0.07) & (0.21)\\
 & RMSE & 0.07 & 0.46 & 0.18 & 5.55 & 0.07 & 0.21\\
 & $b_3$ & 0. & -0.33 & -0.13 & 0.2 & 0.01 & 0.05\\
 & 0.8 & (0.05) & (0.18) & (0.27) & (4.43) & (0.06) & (0.19)\\
 & RMSE & 0.05 & 0.37 & 0.3 & 4.41 & 0.06 & 0.19\\
 & GOF  & nc & 0.243 & 0.374 & 0.247 & 0.137 & 0.239\\
 & time  & 0.6 & 61. & 9.4 & 19.2 & 167.6 & 226.6\\
300 & fails  & 0 & 0 & 0 & 0 & 4 & 1\\
\hline
\hline
    \end{tabular}}
    \label{tab:1}
\end{table}
```
```
49 502.208485
```

In[ ]:=
```
ST = SemTest[Nsimu, {100 , 300},
  (First@SimData[#, para0, Spara0, simerr → False, latnorm → 0, errnorm → 0]) &,
  fastM, slowM, {b1, b2, b3}, {1.2, 0.5, 0.8},
  {Bobserved, Blvars, BollenEQ}, LatInit → {9, 1, 5}, bollen → True];
Print@ST["ltx"]; Print[AbsoluteTime[] - t0]
```

```
------------------------------------------------------ 100
```

Method	{b1, b2, b3}	std	#fails	avg time	avg GOF	SD GOF
lavaan	{1.199, 0.492, 0.803}	{0.036, 0.143, 0.121}	0	0.4	{}	{}
W1	{1.196, 0.89, 0.484}	{0.038, 0.464, 0.376}	0	2.5	0.251	0.01
Wn	{1.186, 0.499, 0.8}	{0.035, 0.111, 0.095}	0	1.1	0.252	0.011
Ww(Wn)	{1.176, 0.628, 0.694}	{0.037, 0.155, 0.131}	0	2.2	0.254	0.01
W^{2a}	{1.245, 0.489, 0.814}	{0.036, 0.157, 0.132}	6	25.7	0.152	0.005
W^{2a0}	{1.183, 0.463, 0.832}	{0.038, 0.324, 0.268}	2	30.2	0.24	0.009

```
------------------------------------------------------ 300
```

Method	{b1, b2, b3}	std	#fails	avg time	avg GOF	SD GOF
lavaan	{1.199, 0.492, 0.806}	{0.029, 0.111, 0.094}	0	0.4	{}	{}
W1	{1.196, 0.881, 0.493}	{0.03, 0.359, 0.292}	0	19.8	0.247	0.009
Wn	{1.117, 0.47, 0.758}	{0.268, 0.13, 0.188}	0	3.6	0.295	0.18
Ww(Wn)	{1.18, 0.161, 1.078}	{0.034, 3.695, 2.976}	0	7.4	0.25	0.009
W^{2a}	{1.245, 0.488, 0.817}	{0.029, 0.121, 0.101}	12	78.7	0.145	0.008
W^{2a0}	{1.182, 0.429, 0.863}	{0.032, 0.62, 0.517}	2	75.7	0.238	0.009

```
\begin{table}
         \caption{Titel (100 simulations for each $n$) }
        {\footnotesize\begin{tabular}{cccccccc}\hline
$n$ & var   & ML  & $W_1$  & $W_n$  & $Ww(W_n)$  & $W^{2a}$  & $W^{2a0}$ \\
100 & $b_1$ & 0. & 0. & -0.01 & -0.02 & 0.04 & -0.02\\
 & 1.2 & (0.04) & (0.04) & (0.04) & (0.04) & (0.04) & (0.04)\\
 & RMSE & 0.04 & 0.04 & 0.04 & 0.04 & 0.06 & 0.04\\
 & $b_2$ & -0.01 & 0.39 & 0. & 0.13 & -0.01 & -0.04\\
 & 0.5 & (0.14) & (0.46) & (0.11) & (0.15) & (0.16) & (0.32)\\
 & RMSE & 0.14 & 0.6 & 0.11 & 0.2 & 0.16 & 0.32\\
 & $b_3$ & 0. & -0.32 & 0. & -0.11 & 0.01 & 0.03\\
 & 0.8 & (0.12) & (0.38) & (0.09) & (0.13) & (0.13) & (0.27)\\
 & RMSE & 0.12 & 0.49 & 0.09 & 0.17 & 0.13 & 0.27\\
 & GOF  & nc & 0.251 & 0.252 & 0.254 & 0.152 & 0.24\\
 & time  & 0.4 & 2.5 & 1.1 & 2.2 & 25.7 & 30.2\\
100 & fails  & 0 & 0 & 0 & 0 & 6 & 2\\
\hline
300 & $b_1$ & 0. & 0. & -0.08 & -0.02 & 0.05 & -0.02\\
 & 1.2 & (0.03) & (0.03) & (0.27) & (0.03) & (0.03) & (0.03)\\
 & RMSE & 0.03 & 0.03 & 0.28 & 0.04 & 0.05 & 0.04\\
 & $b_2$ & -0.01 & 0.38 & -0.03 & -0.34 & -0.01 & -0.07\\
 & 0.5 & (0.11) & (0.36) & (0.13) & (3.7) & (0.12) & (0.62)\\
 & RMSE & 0.11 & 0.52 & 0.13 & 3.7 & 0.12 & 0.62\\
 & $b_3$ & 0.01 & -0.31 & -0.04 & 0.28 & 0.02 & 0.06\\
 & 0.8 & (0.09) & (0.29) & (0.19) & (2.98) & (0.1) & (0.52)\\
 & RMSE & 0.09 & 0.42 & 0.19 & 2.98 & 0.1 & 0.52\\
 & GOF  & nc & 0.247 & 0.295 & 0.25 & 0.145 & 0.238\\
 & time  & 0.4 & 19.8 & 3.6 & 7.4 & 78.7 & 75.7\\
300 & fails  & 0 & 0 & 0 & 0 & 12 & 2\\
\hline
\hline
    \end{tabular}}
    \label{tab:1}
\end{table}
```

$3.900269653462977 \times 10^{9} - t0$

In[•]:=
```
ST = SemTest[Nsimu, {100 , 300},
  (First@SimData[#, para0, Spara0b, simerr → False, latnorm → 0, errnorm → 0]) &,
  fastM, slowM, {b1, b2, b3}, {1.2, 0.5, 0.8},
  {Bobserved, Blvars, BollenEQ}, LatInit → {9, 1, 5}, bollen → True];
Print@ST["ltx"]; Print[AbsoluteTime[] - t0]
```

```
------------------------------------------------------ 100
```

Method	{b1, b2, b3}	std	#fails	avg time	avg GOF	SD GOF
lavaan	{1.2, 0.505, 0.798}	{0.052, 0.116, 0.089}	0	0.4	{}	{}
W1	{1.204, 1.058, 0.365}	{0.058, 0.528, 0.409}	0	2.5	0.279	0.014
Wn	{1.174, 0.518, 0.79}	{0.052, 0.109, 0.082}	0	1.1	0.248	0.011
Ww(Wn)	{1.162, 0.565, 0.753}	{0.053, 0.122, 0.094}	0	2.	0.264	0.012
W^{2a}	{1.282, 0.499, 0.819}	{0.055, 0.125, 0.096}	7	26.6	0.162	0.007
W^{2a0}	{1.186, 0.56, 0.758}	{0.06, 0.464, 0.338}	1	31.	0.254	0.01

```
------------------------------------------------------ 300
```

Method	{b1, b2, b3}	std	#fails	avg time	avg GOF	SD GOF
lavaan	{1.199, 0.503, 0.797}	{0.044, 0.092, 0.072}	0	0.5	{}	{}
W1	{1.202, 1.042, 0.374}	{0.049, 0.417, 0.325}	0	21.2	0.275	0.012
Wn	{1.065, 0.473, 0.721}	{0.326, 0.15, 0.217}	0	3.5	0.315	0.22
Ww(Wn)	{1.172, -0.007, 1.204}	{0.056, 3.071, 2.394}	0	7.	0.262	0.015
W^{2a}	{1.278, 0.501, 0.816}	{0.045, 0.1, 0.078}	15	74.4	0.154	0.01
W^{2a0}	{1.183, 0.489, 0.813}	{0.048, 0.53, 0.424}	3	80.1	0.25	0.01

```
\begin{table}
         \caption{Titel (100 simulations for each $n$) }
        {\footnotesize\begin{tabular}{cccccccc}\hline
$n$ & var   & ML  & $W_1$  & $W_n$  & $Ww(W_n)$  & $W^{2a}$  & $W^{2a0}$ \\
100 & $b_1$ & 0. & 0. & -0.03 & -0.04 & 0.08 & -0.01\\
 & 1.2 & (0.05) & (0.06) & (0.05) & (0.05) & (0.05) & (0.06)\\
 & RMSE & 0.05 & 0.06 & 0.06 & 0.07 & 0.1 & 0.06\\
 & $b_2$ & 0. & 0.56 & 0.02 & 0.07 & 0. & 0.06\\
 & 0.5 & (0.12) & (0.53) & (0.11) & (0.12) & (0.13) & (0.46)\\
 & RMSE & 0.12 & 0.77 & 0.11 & 0.14 & 0.12 & 0.47\\
 & $b_3$ & 0. & -0.43 & -0.01 & -0.05 & 0.02 & -0.04\\
 & 0.8 & (0.09) & (0.41) & (0.08) & (0.09) & (0.1) & (0.34)\\
 & RMSE & 0.09 & 0.6 & 0.08 & 0.1 & 0.1 & 0.34\\
 & GOF  & nc & 0.279 & 0.248 & 0.264 & 0.162 & 0.254\\
 & time  & 0.4 & 2.5 & 1.1 & 2. & 26.6 & 31.\\
100 & fails  & 0 & 0 & 0 & 0 & 7 & 1\\
\hline
300 & $b_1$ & 0. & 0. & -0.14 & -0.03 & 0.08 & -0.02\\
 & 1.2 & (0.04) & (0.05) & (0.33) & (0.06) & (0.05) & (0.05)\\
 & RMSE & 0.04 & 0.05 & 0.35 & 0.06 & 0.09 & 0.05\\
 & $b_2$ & 0. & 0.54 & -0.03 & -0.51 & 0. & -0.01\\
 & 0.5 & (0.09) & (0.42) & (0.15) & (3.07) & (0.1) & (0.53)\\
 & RMSE & 0.09 & 0.68 & 0.15 & 3.11 & 0.1 & 0.53\\
 & $b_3$ & 0. & -0.43 & -0.08 & 0.4 & 0.02 & 0.01\\
 & 0.8 & (0.07) & (0.32) & (0.22) & (2.39) & (0.08) & (0.42)\\
 & RMSE & 0.07 & 0.54 & 0.23 & 2.42 & 0.08 & 0.42\\
 & GOF  & nc & 0.275 & 0.315 & 0.262 & 0.154 & 0.25\\
 & time  & 0.5 & 21.2 & 3.5 & 7. & 74.4 & 80.1\\
300 & fails  & 0 & 0 & 0 & 0 & 15 & 3\\
\hline
\hline
    \end{tabular}}
    \label{tab:1}
\end{table}
```

$3.900209530381702 \times 10^{9} - t0$

In[•]:=
```
ST = SemTest[Nsimu, {100 , 200},
  (First@SimData[#, para0, Spara0, simerr → False, latnorm → 1, errnorm → 2]) &,
  fastM, slowM, {b1, b2, b3}, {1.2, 0.5, 0.8},
  {Bobserved, Blvars, BollenEQ}, LatInit → {9, 1, 5}, bollen → True];
Print@ST["ltx"]; Print[AbsoluteTime[] - t0]
```

```
------------------------------------------------------ 100
```

Method	{b1, b2, b3}	std	#fails	avg time	avg GOF	SD GOF
lavaan	{1.195, 0.514, 0.791}	{0.043, 0.129, 0.098}	0	0.4	{}	{}
W1	{1.193, 0.915, 0.466}	{0.046, 0.379, 0.305}	0	2.4	0.249	0.01
Wn	{1.184, 0.511, 0.794}	{0.043, 0.098, 0.075}	0	1.1	0.249	0.011
Ww(Wn)	{1.173, 0.648, 0.681}	{0.044, 0.152, 0.119}	0	2.1	0.252	0.009
W^{2a}	{1.241, 0.515, 0.797}	{0.043, 0.135, 0.103}	5	22.8	0.152	0.005
W^{2a0}	{1.178, 0.47, 0.832}	{0.044, 0.213, 0.172}	1	24.2	0.239	0.008

```
------------------------------------------------------ 200
```

Method	{b1, b2, b3}	std	#fails	avg time	avg GOF	SD GOF
lavaan	{1.2, 0.513, 0.791}	{0.036, 0.112, 0.088}	0	0.4	{}	{}
W1	{1.198, 0.929, 0.455}	{0.039, 0.335, 0.272}	0	6.8	0.247	0.008
Wn	{0.945, 0.416, 0.639}	{0.453, 0.188, 0.297}	0	3.1	0.408	0.3
Ww(Wn)	{1.183, 0.664, 0.667}	{0.039, 0.194, 0.16}	0	4.1	0.249	0.01
W^{2a}	{1.246, 0.512, 0.799}	{0.037, 0.117, 0.092}	11	41.6	0.146	0.007
W^{2a0}	{1.182, 0.468, 0.832}	{0.037, 0.198, 0.163}	3	48.	0.237	0.008

```
\begin{table}
         \caption{Titel (100 simulations for each $n$) }
        {\footnotesize\begin{tabular}{cccccccc}\hline
$n$ & var   & ML  & $W_1$  & $W_n$  & $Ww(W_n)$  & $W^{2a}$  & $W^{2a0}$ \\
100 & $b_1$ & 0. & -0.01 & -0.02 & -0.03 & 0.04 & -0.02\\
 & 1.2 & (0.04) & (0.05) & (0.04) & (0.04) & (0.04) & (0.04)\\
 & RMSE & 0.04 & 0.05 & 0.05 & 0.05 & 0.06 & 0.05\\
 & $b_2$ & 0.01 & 0.41 & 0.01 & 0.15 & 0.02 & -0.03\\
 & 0.5 & (0.13) & (0.38) & (0.1) & (0.15) & (0.14) & (0.21)\\
 & RMSE & 0.13 & 0.56 & 0.1 & 0.21 & 0.14 & 0.21\\
 & $b_3$ & -0.01 & -0.33 & -0.01 & -0.12 & 0. & 0.03\\
 & 0.8 & (0.1) & (0.31) & (0.08) & (0.12) & (0.1) & (0.17)\\
 & RMSE & 0.1 & 0.45 & 0.08 & 0.17 & 0.1 & 0.17\\
 & GOF  & nc & 0.249 & 0.249 & 0.252 & 0.152 & 0.239\\
 & time  & 0.4 & 2.4 & 1.1 & 2.1 & 22.8 & 24.2\\
100 & fails  & 0 & 0 & 0 & 0 & 5 & 1\\
\hline
200 & $b_1$ & 0. & 0. & -0.26 & -0.02 & 0.05 & -0.02\\
 & 1.2 & (0.04) & (0.04) & (0.45) & (0.04) & (0.04) & (0.04)\\
 & RMSE & 0.04 & 0.04 & 0.52 & 0.04 & 0.06 & 0.04\\
 & $b_2$ & 0.01 & 0.43 & -0.08 & 0.16 & 0.01 & -0.03\\
 & 0.5 & (0.11) & (0.34) & (0.19) & (0.19) & (0.12) & (0.2)\\
 & RMSE & 0.11 & 0.54 & 0.21 & 0.25 & 0.12 & 0.2\\
 & $b_3$ & -0.01 & -0.35 & -0.16 & -0.13 & 0. & 0.03\\
 & 0.8 & (0.09) & (0.27) & (0.3) & (0.16) & (0.09) & (0.16)\\
 & RMSE & 0.09 & 0.44 & 0.34 & 0.21 & 0.09 & 0.17\\
 & GOF  & nc & 0.247 & 0.408 & 0.249 & 0.146 & 0.237\\
 & time  & 0.4 & 6.8 & 3.1 & 4.1 & 41.6 & 48.\\
200 & fails  & 0 & 0 & 0 & 0 & 11 & 3\\
\hline
\hline
    \end{tabular}}
    \label{tab:1}
\end{table}
```

$3.900231672412485 \times 10^{9} - t0$

```
(* ********************************** *)

In[*]:= (* correlated errors *) ST = SemTest[Nsimu, {100, 300},
  (First@SimData[#, para0, Spara0, simerr → True, latnorm → 0, errnorm → 0]) &,
  fastM, slowM, {b1, b2, b3}, {1.2, 0.5, 0.8}, {Bobserved, Blvars, BollenEQ},
  LatInit → {9, 1, 5}, bollen → True, printSummary → True]; Print@ST["ltx"]
```

```
------------------------------------------------------ 100
```

Method	{b1, b2, b3}	std	#fails	avg time	avg GOF	SD GOF
lavaan	{1.205, 0.476, 0.819}	{0.038, 0.128, 0.098}	0	0.4	{}	{}
W1	{1.195, 0.791, 0.568}	{0.042, 0.397, 0.311}	0	2.4	0.275	0.014
Wn	{1.184, 0.47, 0.826}	{0.039, 0.1, 0.073}	0	1.	0.281	0.014
Ww(Wn)	{1.17, 0.553, 0.762}	{0.039, 0.134, 0.101}	0	2.	0.278	0.013
W^{2a}	{1.25, 0.48, 0.823}	{0.042, 0.135, 0.104}	5	25.7	0.158	0.006
W^{2a0}	{1.19, 0.353, 0.925}	{0.038, 0.192, 0.156}	3	23.5	0.242	0.01

```
------------------------------------------------------ 300
```

Method	{b1, b2, b3}	std	#fails	avg time	avg GOF	SD GOF
lavaan	{1.201, 0.479, 0.817}	{0.031, 0.103, 0.08}	0	0.4	{}	{}
W1	{1.191, 0.76, 0.593}	{0.035, 0.309, 0.244}	0	18.	0.272	0.011
Wn	{1.079, 0.433, 0.758}	{0.32, 0.133, 0.221}	0	3.4	0.345	0.204
Ww(Wn)	{1.172, 0.446, 0.847}	{0.038, 2.697, 2.179}	0	7.1	0.274	0.013
W^{2a}	{1.247, 0.482, 0.822}	{0.034, 0.109, 0.085}	8	90.6	0.152	0.008
W^{2a0}	{1.187, 0.277, 0.987}	{0.034, 1.721, 1.381}	3	72.5	0.241	0.009

```
\begin{table}
         \caption{Titel (100 simulations for each $n$) }
        {\footnotesize\begin{tabular}{cccccccc}\hline
$n$ & var   & ML  & $W_1$  & $W_n$  & $Ww(W_n)$  & $W^{2a}$  & $W^{2a0}$ \\
100 & $b_1$ & 0.01 & 0. & -0.02 & -0.03 & 0.05 & -0.01\\
 & 1.2 & (0.04) & (0.04) & (0.04) & (0.04) & (0.04) & (0.04)\\
 & RMSE & 0.04 & 0.04 & 0.04 & 0.05 & 0.06 & 0.04\\
 & $b_2$ & -0.02 & 0.29 & -0.03 & 0.05 & -0.02 & -0.15\\
 & 0.5 & (0.13) & (0.4) & (0.1) & (0.13) & (0.13) & (0.19)\\
 & RMSE & 0.13 & 0.49 & 0.1 & 0.14 & 0.14 & 0.24\\
 & $b_3$ & 0.02 & -0.23 & 0.03 & -0.04 & 0.02 & 0.13\\
 & 0.8 & (0.1) & (0.31) & (0.07) & (0.1) & (0.1) & (0.16)\\
 & RMSE & 0.1 & 0.39 & 0.08 & 0.11 & 0.11 & 0.2\\
 & GOF  & nc & 0.275 & 0.281 & 0.278 & 0.158 & 0.242\\
 & time  & 0.4 & 2.4 & 1. & 2. & 25.7 & 23.5\\
100 & fails  & 0 & 0 & 0 & 0 & 5 & 3\\
\hline
300 & $b_1$ & 0. & -0.01 & -0.12 & -0.03 & 0.05 & -0.01\\
 & 1.2 & (0.03) & (0.03) & (0.32) & (0.04) & (0.03) & (0.03)\\
 & RMSE & 0.03 & 0.04 & 0.34 & 0.05 & 0.06 & 0.04\\
 & $b_2$ & -0.02 & 0.26 & -0.07 & -0.05 & -0.02 & -0.22\\
 & 0.5 & (0.1) & (0.31) & (0.13) & (2.7) & (0.11) & (1.72)\\
 & RMSE & 0.11 & 0.4 & 0.15 & 2.69 & 0.11 & 1.73\\
 & $b_3$ & 0.02 & -0.21 & -0.04 & 0.05 & 0.02 & 0.19\\
 & 0.8 & (0.08) & (0.24) & (0.22) & (2.18) & (0.08) & (1.38)\\
 & RMSE & 0.08 & 0.32 & 0.22 & 2.17 & 0.09 & 1.39\\
 & GOF  & nc & 0.272 & 0.345 & 0.274 & 0.152 & 0.241\\
 & time  & 0.4 & 18. & 3.4 & 7.1 & 90.6 & 72.5\\
300 & fails  & 0 & 0 & 0 & 0 & 8 & 3\\
\hline
\hline
    \end{tabular}}
    \label{tab:1}
\end{table}
```

In[ ]:=
```
(*************** Nonlin Variant of Bollen's model *)
BNLEQ = {{dem60 == b1 * ind60 + u1, e01}, {dem65 == b2 * ind60 + b3 * dem60^2 + u2, e02},
  {x1 == 1 * ind60 + t1, ee1}, {x2 == c2 * ind60 + t2, ee2}, {x3 == c3 * ind60 + t3, ee3},
  {y1 == 1 * dem60 + s1, e1}, {y2 == d2 * dem60 + s2, e2}, {y3 == d3 * dem60 + s3, e3},
  {y4 == d4 * dem60 + s4, e4}, {y5 == 1 * dem65 + s5, e5}, {y6 == d6 * dem65 + s6, e6},
  {y7 == d7 * dem65 + s7, e7}, {y8 == d8 * dem65 + s8, e8} };
Options[SimDataNL] = {simerr → False, latnorm → True, errnorm → True};
SimDataNL[n_, {pb1_, pb2_, pb3_, pc2_, pc3_, pd2_, pd3_, pd4_, pd6_, pd7_, pd8_},
   {sigX1_, sigX2_, sigX3_, sigY1_, sigY2_, sigY3_, sigY4_,
    sigY5_, sigY6_, sigY7_, sigY8_, sig1_, sig2_}, OptionsPattern[]] :=
  Module[{i, IND60, DEM60, DEM60a, k, ksol, DEM65, X1, X2, X3, Y1, Y2, Y3, Y4, Y5, Y6,
    Y7, Y8, res, ee, EVR, E01, E02, EE1, EE2, EE3, E1, E2, E3, E4, E5, E6, E7, E8},
   EVR = If[OptionValue@errnorm, NVR, UVR];
   IND60 = If[OptionValue@latnorm, NVR[1, n], UVR[1, n]];
   X1 = 1.0 * IND60 + EVR[sigX1, n]; EE1 = X1 - IND60;
   X2 = pc2 * IND60 + EVR[sigX2, n]; EE2 = X2 - pc2 * IND60;
   X3 = pc3 * IND60 + EVR[sigX3, n]; EE3 = X3 - pc3 * IND60;
   DEM60 = pb1 * IND60 + EVR[sig1, n]; E01 = DEM60 - pb1 * IND60;
   Y1 = 1.0 * DEM60 + EVR[sigY1, n]; E1 = Y1 - DEM60;
   Y2 = pd2 * DEM60 + EVR[sigY2, n]; E2 = Y2 - pd2 * DEM60;
   Y3 = pd3 * DEM60 + EVR[sigY3, n]; E3 = Y3 - pd3 * DEM60;
   Y4 = pd4 * DEM60 + EVR[sigY4, n]; E4 = Y4 - pd4 * DEM60;
   DEM65 = pb2 * IND60 + pb3 * DEM60^2 + NVR[sig2, n];
   E02 = DEM65 - pb2 * IND60 - pb3 * IND60 * DEM60;
   Y5 = 1.0 * DEM65 + EVR[sigY5, n]; E5 = Y5 - 1 * DEM65;
   Y6 = pd6 * DEM65 + EVR[sigY6, n]; E6 = Y6 - pd6 * DEM65;
   Y7 = pd7 * DEM65 + EVR[sigY7, n]; E7 = Y7 - pd7 * DEM65;
   Y8 = pd8 * DEM65 + EVR[sigY8, n]; E8 = Y8 - pd8 * DEM65;
   If[OptionValue@simerr, ee = NVR[0.3, n];
    Y4 = Y4 + ee;
    Y8 = Y8 + ee;
    E4 = E4 + ee;
    E8 = E8 + ee;];
   res = Transpose[{Y1, Y2, Y3, Y4, Y5, Y6, Y7, Y8, X1, X2, X3}];
   {res, Map[Apply[Rule, #] &,
     Join[{{b1, pb1}, {b2, pb2}, {b3, pb3}, {c2, pc2}, {c3, pc3},
       {d2, pd2}, {d3, pd3}, {d4, pd4}, {d6, pd6}, {d7, pd7}, {d8, pd8}},
      Table[{ind60[i], IND60[[i]]}, {i, 1, n}], Table[{dem60[i], DEM60[[i]]},
       {i, 1, n}], Table[{dem65[i], DEM65[[i]]}, {i, 1, n}]]],
    Map[StandardDeviation, {IND60, DEM60, DEM65}],
    {E01, E02, EE1, EE2, EE3, E1, E2, E3, E4, E5, E6, E7, E8}}];
ST = SemTest[Nsimu, {100, 300}, (First@SimDataNL[#, para0,
      Spara0, simerr → False, latnorm → False, errnorm → False]) &,
  fastM, slowM,
  {b1, b2, b3}, {1.2, 0.5, 0.8}, {Bobserved, Blvars, BNLEQ}, bollen → False,
  LatInit → {9, 1, 5}, printSummary → True, LatentMean0 → True]; Print@ST["ltx"]
```

```
------------------------------------------------------- 100
```

Method	{b1, b2, b3}	std	#fails	avg time	avg GOF	SD GOF
W1	{1.201, 0.445, 0.836}	{0.045, 0.246, 0.039}	0	1.6	0.255	0.012
Wn	{1.196, 0.444, 0.782}	{0.043, 0.237, 0.037}	0	1.6	0.266	0.013
Ww(Wn)	{1.187, 0.441, 0.805}	{0.044, 0.238, 0.038}	0	2.7	0.262	0.013
W^{2a}	{1.25, 0.435, 0.824}	{0.042, 0.265, 0.037}	5	43.8	0.171	0.012
W^{2a0}	{1.188, 0.446, 0.837}	{0.044, 0.243, 0.042}	1	34.8	0.243	0.014

```
------------------------------------------------------- 300
```

Method	{b1, b2, b3}	std	#fails	avg time	avg GOF	SD GOF
W1	{1.198, 0.47, 0.837}	{0.035, 0.197, 0.032}	0	4.6	0.243	0.015
Wn	{1.194, 0.467, 0.78}	{0.034, 0.189, 0.03}	0	4.7	0.252	0.017
Ww(Wn)	{1.184, 0.465, 0.805}	{0.035, 0.19, 0.031}	0	8.5	0.249	0.016
W^{2a}	{1.247, 0.465, 0.825}	{0.033, 0.212, 0.03}	8	123.9	0.158	0.017
W^{2a0}	{1.183, 0.471, 0.841}	{0.035, 0.194, 0.035}	3	90.5	0.231	0.019

```
\begin{table}
         \caption{Titel (100 simulations for each $n$) }
        {\footnotesize\begin{tabular}{ccccccc}\hline
$n$ & var   & $W_1$  & $W_n$  & $Ww(W_n)$  & $W^{2a}$  & $W^{2a0}$ \\
100 & $b_1$ & 0. & 0. & -0.01 & 0.05 & -0.01\\
 & 1.2 & (0.04) & (0.04) & (0.04) & (0.04) & (0.04)\\
 & RMSE & 0.04 & 0.04 & 0.05 & 0.06 & 0.05\\
 & $b_2$ & -0.06 & -0.06 & -0.06 & -0.06 & -0.05\\
 & 0.5 & (0.25) & (0.24) & (0.24) & (0.26) & (0.24)\\
 & RMSE & 0.25 & 0.24 & 0.24 & 0.27 & 0.25\\
 & $b_3$ & 0.04 & -0.02 & 0. & 0.02 & 0.04\\
 & 0.8 & (0.04) & (0.04) & (0.04) & (0.04) & (0.04)\\
 & RMSE & 0.05 & 0.04 & 0.04 & 0.04 & 0.06\\
 & GOF  & 0.255 & 0.266 & 0.262 & 0.171 & 0.243\\
 & time  & 1.6 & 1.6 & 2.7 & 43.8 & 34.8\\
100 & fails  & 0 & 0 & 0 & 5 & 1\\
\hline
300 & $b_1$ & 0. & -0.01 & -0.02 & 0.05 & -0.02\\
 & 1.2 & (0.04) & (0.03) & (0.04) & (0.03) & (0.04)\\
 & RMSE & 0.04 & 0.03 & 0.04 & 0.06 & 0.04\\
 & $b_2$ & -0.03 & -0.03 & -0.03 & -0.04 & -0.03\\
 & 0.5 & (0.2) & (0.19) & (0.19) & (0.21) & (0.19)\\
 & RMSE & 0.2 & 0.19 & 0.19 & 0.21 & 0.2\\
 & $b_3$ & 0.04 & -0.02 & 0.01 & 0.02 & 0.04\\
 & 0.8 & (0.03) & (0.03) & (0.03) & (0.03) & (0.04)\\
 & RMSE & 0.05 & 0.04 & 0.03 & 0.04 & 0.05\\
 & GOF  & 0.243 & 0.252 & 0.249 & 0.158 & 0.231\\
 & time  & 4.6 & 4.7 & 8.5 & 123.9 & 90.5\\
300 & fails  & 0 & 0 & 0 & 8 & 3\\
\hline
\hline
    \end{tabular}}
    \label{tab:1}
\end{table}
```

```
In[•]:= ST = SemTest[50, {1000}, (First@SimDataNL[#, para0,
          Spara0b, simerr → False, latnorm → False, errnorm → False]) &,
        fastM, slowM,
        {b1, b2, b3}, {1.2, 0.5, 0.8}, {Bobserved, Blvars, BNLEQ},
        bollen → False, printSummary → True]; Print@ST["ltx"]

In[•]:= (************ Ganzachs model  **************)
      modelG =
        {{y1 == 1 * eta + oy1, d1}, {y2 == ly12 * eta + oy2, d2}, {y3 == ly13 * eta + oy3, d3},
         {x1 == 1 * xi1 + ox1, e1}, {x2 == lx12 * xi1 + ox2, e2}, {x3 == lx13 * xi1 + ox3, e3},
         {x4 == 1 * xi2 + ox4, e4}, {x5 == lx52 * xi2 + ox5, e5}, {x6 == lx62 * xi2 + ox6, e6},
         {eta == om11 * xi1^2 + om12 * xi1 * xi2 +
            om22 * xi2^2 + gamma1 * xi1 + gamma2 * xi2 + oeta, e0}};
      Gtrue = {0.5, 0.3, 0.2, 0.3, 0.2};
      Gpara = {om11, om12, om22, gamma1, gamma2};
      Gsimudat[n_] := Gsimudat[n, 0.2, 0.3, 0.2, 0.1, 0.3, 0.4, 0.3, 0.1, 0.3, 0.1];
      Gsimudat[n_,              sn_, sx1_, sx2_, sx3_, sx4_, sx5_, sx6_, sy1_, sy2_,
        sy3_] := Module[{XI, XI1, XI2, Etas, xs1, xs2, xs3, xs4, xs5, xs6, ys1, ys2, ys3},
        XI = RandomVariate[MultinormalDistribution[{0, 0}, {{1, 0.3}, {0.3, 1}}], n];
        XI1 = Transpose[XI][[1]]; XI2 = Transpose[XI][[2]];
        Etas = Gtrue[[1]] * XI1^2 + Gtrue[[2]] * XI1 * XI2 + Gtrue[[3]] * XI2^2 + Gtrue[[4]] * XI1 +
          Gtrue[[5]] * XI2 + RandomVariate[NormalDistribution[0, sn], n];
      xs1 = 1 * XI1 + RandomVariate[NormalDistribution[0, sx1], n];
        xs2 = 0.7 * XI1 + RandomVariate[NormalDistribution[0, sx2], n];
      xs3 = 1.2 * XI1 + RandomVariate[NormalDistribution[0, sx3], n];
        xs4 = 1 * XI2 + RandomVariate[NormalDistribution[0, sx4], n];
      xs5 = 0.5 * XI2 + RandomVariate[NormalDistribution[0, sx5], n];
        xs6 = 0.9 * XI2 + RandomVariate[NormalDistribution[0, sx6], n];
      ys1 = 1 * Etas + RandomVariate[NormalDistribution[0, sy1], n];
        ys2 = 0.8 * Etas + RandomVariate[NormalDistribution[0, sy2], n];
      ys3 = 1.3 * Etas + RandomVariate[NormalDistribution[0, sy3], n];
      Transpose[{xs1, xs2, xs3, xs4, xs5, xs6, ys1, ys2, ys3}]];
      Gobs = {x1, x2, x3, x4, x5, x6, y1, y2, y3};
      ST = SemTest[Nsimu / 10, {100, 300}, Gsimudat, fastM, slowM, Gpara, Gtrue,
        {Gobs, {xi1, xi2, eta}, modelG}, LatInit → {1, 4, 7}]; Print@ST["ltx"]
```

```
\begin{table}
         \caption{Titel (10 simulations for each $n$) }
        {\footnotesize\begin{tabular}{ccccccc}\hline
$n$ & var   & $W_1$  & $W_n$  & $Ww(W_n)$  & $W^{2a}$  & $W^{2a0}$ \\
100 & $om11$ & -0.01 & -0.02 & 0. & 0.03 & 0.02\\
 & 0.5 & (0.05) & (0.03) & (0.04) & (0.05) & (0.06)\\
 & RMSE & 0.04 & 0.04 & 0.04 & 0.06 & 0.06\\
 & $om12$ & -0.02 & -0.02 & -0.01 & 0.02 & 0.01\\
 & 0.3 & (0.04) & (0.03) & (0.03) & (0.23) & (0.03)\\
 & RMSE & 0.04 & 0.03 & 0.03 & 0.22 & 0.03\\
 & $om22$ & -0.01 & -0.01 & 0. & 0.03 & 0.03\\
 & 0.2 & (0.02) & (0.02) & (0.02) & (0.17) & (0.03)\\
 & RMSE & 0.02 & 0.02 & 0.02 & 0.16 & 0.04\\
 & $gamma1$ & -0.06 & -0.04 & -0.03 & -0.13 & -0.09\\
 & 0.3 & (0.27) & (0.06) & (0.05) & (0.59) & (0.26)\\
 & RMSE & 0.26 & 0.07 & 0.06 & 0.58 & 0.26\\
 & $gamma2$ & -0.01 & -0.01 & -0.01 & -0.12 & 0.02\\
 & 0.2 & (0.13) & (0.03) & (0.03) & (0.45) & (0.23)\\
 & RMSE & 0.12 & 0.03 & 0.03 & 0.44 & 0.22\\
 & GOF  & 0.307 & 0.315 & 0.293 & 0.214 & 0.273\\
 & time  & 1.3 & 0.8 & 1.7 & 53.6 & 18.9\\
100 & fails  & 0 & 0 & 0 & 0 & 1\\
\hline
300 & $om11$ & -0.02 & -0.03 & -0.01 & 0.02 & 0.01\\
 & 0.5 & (0.04) & (0.03) & (0.03) & (0.04) & (0.05)\\
 & RMSE & 0.04 & 0.04 & 0.03 & 0.05 & 0.05\\
 & $om12$ & -0.01 & -0.01 & 0. & 0. & 0.01\\
 & 0.3 & (0.03) & (0.03) & (0.03) & (0.16) & (0.03)\\
 & RMSE & 0.03 & 0.03 & 0.03 & 0.16 & 0.03\\
 & $om22$ & -0.01 & -0.02 & -0.01 & 0.04 & 0.02\\
 & 0.2 & (0.02) & (0.02) & (0.02) & (0.11) & (0.03)\\
 & RMSE & 0.02 & 0.02 & 0.02 & 0.12 & 0.03\\
 & $gamma1$ & 0. & -0.03 & -0.02 & 0. & 0.16\\
 & 0.3 & (0.36) & (0.06) & (0.06) & (0.47) & (1.01)\\
 & RMSE & 0.35 & 0.07 & 0.06 & 0.46 & 1.\\
 & $gamma2$ & -0.01 & -0.01 & -0.01 & -0.05 & 0.18\\
 & 0.2 & (0.17) & (0.04) & (0.04) & (0.34) & (0.94)\\
 & RMSE & 0.17 & 0.04 & 0.04 & 0.33 & 0.93\\
 & GOF  & 0.295 & 0.302 & 0.278 & 0.193 & 0.255\\
 & time  & 8.3 & 2.9 & 6. & 148.8 & 104.\\
300 & fails  & 0 & 0 & 0 & 0 & 1\\
\hline
\hline
    \end{tabular}}
    \label{tab:1}
\end{table}
```

```
 Method   {om11, om12, om22, gamma1, gamma2}                 std                 #fails a
   W1    {0.485, 0.291, 0.187, 0.304, 0.192} {0.036, 0.034, 0.018, 0.362, 0.172}    0
   Wn    {0.472, 0.291, 0.184, 0.274, 0.192} {0.028, 0.031, 0.019, 0.063, 0.039}    0
 Ww(Wn)   {0.491, 0.295, 0.188, 0.28, 0.191}  {0.031, 0.03, 0.017, 0.058, 0.037}    0
 W^{2a}  {0.524, 0.297, 0.236, 0.302, 0.148} {0.044, 0.162, 0.114, 0.468, 0.337}    0
W^{2a0}  {0.512, 0.311, 0.223, 0.46, 0.383}  {0.046, 0.031, 0.026, 1.015, 0.936}    1

------------------------------------------------------- 300
 Method   {om11, om12, om22, gamma1, gamma2}                 std                 #fails a
   W1    {0.492, 0.281, 0.194, 0.237, 0.186}  {0.045, 0.036, 0.02, 0.268, 0.129}    0
   Wn     {0.48, 0.283, 0.19, 0.261, 0.191}   {0.032, 0.029, 0.02, 0.058, 0.034}    0
 Ww(Wn)  {0.499, 0.288, 0.196, 0.268, 0.187} {0.039, 0.029, 0.018, 0.051, 0.031}    0
 W^{2a}  {0.532, 0.321, 0.231, 0.175, 0.078} {0.055, 0.231, 0.165, 0.593, 0.446}    0
W^{2a0}  {0.523, 0.309, 0.229, 0.212, 0.223} {0.061, 0.031, 0.034, 0.263, 0.234}    1

------------------------------------------------------- 100
```

```
(* Linear regression  - not identified model  ******************)
```

In[•]:=

```
RegSimu1[n_, RegLatNorm_, sig_, sigx_, sigy_] :=
  Module[{ X00 = If[RegLatNorm, RandomVariate[NormalDistribution[0, sig], n],
      RandomVariate[UniformDistribution[{-2 * sig, 2 * sig}], n]],
    x1o, y1o},
   x1o = X00 + RandomVariate[NormalDistribution[0, sigx], n];
   y1o = 0.5 * X00 + RandomVariate[NormalDistribution[0, sigy], n];
   Transpose[{x1o, y1o}]];
rmod = {{x1 == X0 + u1, e1}, {y1 == a * X0 + u2, e2}};
(*ST=SemTest[10+0*Nsimu,{100,300},RegSimu1[#,True,1,0.4,0.2]&,fastM,slowM,
  {a},{0.5},{{x1,y1},{X0},rmod},LatInit→{1},printSummary→True];
Print@ST["ltx"];*)
(* Now again with non-normal latent *)
ST = SemTest[Nsimu, {100, 300}, RegSimu1[#, False, 1, 0.4, 0.2] &,
  fastM, slowM,
  {a}, {0.5}, {{x1, y1}, {X0}, rmod}, LatInit → {1}, printSummary → True];
Print@ST["ltx"];
```

```
------------------------------------------------------- 100
```

Method	{a}	std	#fails	avg time	avg GOF	SD GOF
W1	{0.465}	{0.023}	0	0.1	0.456	0.01
Wn	{0.465}	{0.023}	0	0.1	0.456	0.01
Ww(Wn)	{0.451}	{0.022}	0	0.2	0.475	0.011
W^{2a}	{0.545}	{0.204}	37	7.7	0.52	0.121
W^{2a0}	{0.483}	{0.064}	23	4.	0.487	0.078

```
------------------------------------------------------- 300
```

Method	{a}	std	#fails	avg time	avg GOF	SD GOF
W1	{0.465}	{0.018}	0	0.4	0.457	0.008
Wn	{0.465}	{0.018}	0	0.2	0.457	0.008
Ww(Wn)	{0.451}	{0.018}	0	0.4	0.477	0.009
W^{2a}	{0.537}	{0.192}	74	17.1	0.519	0.121
W^{2a0}	{0.482}	{0.06}	48	8.7	0.486	0.066

```
\begin{table}
         \caption{Titel (100 simulations for each $n$) }
        {\footnotesize\begin{tabular}{ccccccc}\hline
$n$ & var   & $W_1$  & $W_n$  & $Ww(W_n)$  & $W^{2a}$  & $W^{2a0}$ \\
100 & $a$ & -0.03 & -0.03 & -0.05 & 0.05 & -0.02\\
 & 0.5 & (0.02) & (0.02) & (0.02) & (0.2) & (0.06)\\
 & RMSE & 0.04 & 0.04 & 0.05 & 0.21 & 0.07\\
 & GOF  & 0.456 & 0.456 & 0.475 & 0.52 & 0.487\\
 & time  & 0.1 & 0.1 & 0.2 & 7.7 & 4.\\
100 & fails  & 0 & 0 & 0 & 37 & 23\\
\hline
300 & $a$ & -0.03 & -0.03 & -0.05 & 0.04 & -0.02\\
 & 0.5 & (0.02) & (0.02) & (0.02) & (0.19) & (0.06)\\
 & RMSE & 0.04 & 0.04 & 0.05 & 0.19 & 0.06\\
 & GOF  & 0.457 & 0.457 & 0.477 & 0.519 & 0.486\\
 & time  & 0.4 & 0.2 & 0.4 & 17.1 & 8.7\\
300 & fails  & 0 & 0 & 0 & 74 & 48\\
\hline
\hline
    \end{tabular}}
    \label{tab:1}
\end{table}
```

In[•]:=
```
(* Linear regression  with 2 indicators each (identified)  *)
RegSimu2[n_, RegLatNorm_, sig_, sigx_, sigy_] :=
  Module[{ X00 = If[RegLatNorm, RandomVariate[NormalDistribution[0, sig], n],
       RandomVariate[UniformDistribution[{-2 * sig, 2 * sig}], n]],
    x1o, y1o, x2o, y2o},
   x1o = X00 + RandomVariate[NormalDistribution[0, sigx], n];
   y1o = 0.5 * X00 + RandomVariate[NormalDistribution[0, sigy], n];
   x2o = X00 + RandomVariate[NormalDistribution[0, sigx], n];
   y2o = 0.5 * X00 + RandomVariate[NormalDistribution[0, sigy], n];
   Transpose[{x1o, x2o, y1o, y2o}]];
rmod2 =
  {{x1 == X0 + u1, e1}, {y1 == a * X0 + u2, e2}, {x2 == X0 + u3, e3}, {y2 == a * X0 + u4, e4}};
ST = SemTest[10 + 0 * Nsimu, {100, 300}, RegSimu2[#, True, 1, 0.4, 0.2] &,
  fastM, slowM, {a}, {0.5}, {{x1, x2, y1, y2}, {X0}, rmod2},
  LatInit → {1}, printSummary → True]; Print@ST["ltx"];
(* Again with non-normal latent *)
ST = SemTest[Nsimu, {100, 300}, RegSimu2[#, False, 1, 0.4, 0.2] &,
  fastM, slowM, {a}, {0.5}, {{x1, x2, y1, y2}, {X0}, rmod2},
  LatInit → {1}, printSummary → True]; Print@ST["ltx"]
```

```
------------------------------------------------------ 100
```

Method	{a}	std	#fails	avg time	avg GOF	SD GOF
W1	{0.481}	{0.021}	0	0.2	0.456	0.026
Wn	{0.481}	{0.021}	0	0.2	0.456	0.026
Ww(Wn)	{0.489}	{0.021}	0	0.3	0.432	0.023
W^{2a}	{0.502}	{0.022}	3	1.8	0.364	0.018
W^{2a0}	{0.504}	{0.032}	4	1.9	0.439	0.055

```
------------------------------------------------------ 300
```

Method	{a}	std	#fails	avg time	avg GOF	SD GOF
W1	{0.478}	{0.018}	0	0.6	0.446	0.021
Wn	{0.478}	{0.018}	0	0.4	0.446	0.021
Ww(Wn)	{0.487}	{0.019}	0	0.9	0.421	0.02
W^{2a}	{0.501}	{0.019}	5	10.4	0.352	0.017
W^{2a0}	{0.499}	{0.026}	6	4.7	0.414	0.048

```
\begin{table}
         \caption{Titel (10 simulations for each $n$) }
        {\footnotesize\begin{tabular}{ccccccc}\hline
$n$ & var   & $W_1$  & $W_n$  & $Ww(W_n)$  & $W^{2a}$  & $W^{2a0}$ \\
100 & $a$ & -0.02 & -0.02 & -0.01 & 0. & 0.\\
 & 0.5 & (0.02) & (0.02) & (0.02) & (0.02) & (0.03)\\
 & RMSE & 0.03 & 0.03 & 0.02 & 0.02 & 0.03\\
 & GOF  & 0.456 & 0.456 & 0.432 & 0.364 & 0.439\\
 & time  & 0.2 & 0.2 & 0.3 & 1.8 & 1.9\\
100 & fails  & 0 & 0 & 0 & 3 & 4\\
\hline
300 & $a$ & -0.02 & -0.02 & -0.01 & 0. & 0.\\
 & 0.5 & (0.02) & (0.02) & (0.02) & (0.02) & (0.03)\\
 & RMSE & 0.03 & 0.03 & 0.02 & 0.02 & 0.03\\
 & GOF  & 0.446 & 0.446 & 0.421 & 0.352 & 0.414\\
 & time  & 0.6 & 0.4 & 0.9 & 10.4 & 4.7\\
300 & fails  & 0 & 0 & 0 & 5 & 6\\
\hline
\hline
    \end{tabular}}
    \label{tab:1}
\end{table}
```

(******** Implications **********)**

In[151]:=

```
PosId[x_] := (1 / 2 + 1 / Pi * ArcTan[500 * x]);(* with implication *)
imodel1 = {{x1 == X0, e1}, {x2 == c2 * X0 + u1, e2}, {y1 == Y0, e3},
    {y2 == c4 * Y0 + u3, e4}, {PosId[X0] * (Y0 - a * X0) == 0, e5}};
(*without impliation*)
imodel0 = {{x1 == X0, e1}, {x2 == c2 * X0 + u1, e2},
    {y1 == Y0, e3}, {y2 == c4 * Y0 + u3, e4}, {Y0 - a * X0 == 0, e5}};
imodel0b = {{x1 == X0, e1}, {x2 == c2 * X0 + u1, e2},
    {y1 == Y0, e3}, {y2 == c4 * Y0 + u3, e4}, {Y0 - a * X0 - c0 == 0, e5}};
(*with implication wrong direction*)
imodel2 = {{x1 == X0, e1}, {x2 == c2 * X0 + u1, e2}, {y1 == Y0, e3},
    {y2 == c4 * Y0 + u3, e4}, {PosId[Y0] * (Y0 - a * X0) == 0, e5}};
Itrue = {0.7, 0.4, 0.9}; Iparas = {c2, c4, a};
isim[n_] := Module[{X0, Y0, x1, x2, y1, y2},
    X0 = RandomVariate[UniformDistribution[{-1, 1}], n];
    Y0 = Table[If[X0[[i]] < 0, RandomReal[{-1, 1}],
        0.9 * X0[[i]] + RandomReal[{-0.1, 0.1}]], {i, 1, n}];
    x1 = X0 + RandomVariate[NormalDistribution[0, 0.3], n];
    x2 = 0.7 * X0 + RandomVariate[NormalDistribution[0, 0.2], n];
y1 = Y0 + RandomVariate[NormalDistribution[0, 0.1], n];
y2 = 0.4 * Y0 + RandomVariate[NormalDistribution[0, 0.3], n];
Transpose[{x1, x2, y1, y2}]];
```

In[158]:=

```
Print["***** Error Means and SDs of equation between X,Y, tested with W1"];
rese = {};
ress = {};
resa = {};
resg = {};
resf = {};
resgofmc = {};
kmax = 25;
n = 200;
Do[datR = isim[n];
  r0 = fastSEM["Ww(Wn)", datR, {x1, x2, y1, y2},
    {X0, Y0}, imodel0b, LatentMean0 → False, Quiet → True];
  r1 = fastSEM["Ww(Wn)", datR, {x1, x2, y1, y2}, {X0, Y0},
    imodel1, LatentMean0 → False, Quiet → True];
  r2 = fastSEM["Ww(Wn)", datR, {x1, x2, y1, y2}, {X0, Y0},
    imodel2, LatentMean0 → False, Quiet → True];
  AppendTo[resf, {r0@"Fmin", r1@"Fmin", r2@"Fmin"}];
  AppendTo[rese, {Last@r0["EMeans"], Last@r1["EMeans"], Last@r2["EMeans"]}];
  AppendTo[ress,
   {Sqrt@Last@r0["EVars"], Sqrt@Last@r1["EVars"], Sqrt@Last@r2["EVars"]}];
  AppendTo[resa, {a /. r0["sol"], a /. r1["sol"], a /. r2["sol"]}];
  AppendTo[resgofmc,
   {First@r0["GOFMC"], First@r1["GOFMC"], First@r2["GOFMC"]}];
  AppendTo[resg, {r0["GOF1"], r1["GOF1"], r2["GOF1"]}],
```

```
  {k, kmax}];
Print[{{"", "Corel", "Impl", "false Impl"},
    Join[{"Fmin"}, Round[Map[Mean, Transpose@resf], 0.001]],
    Join[{"M"}, Round[Map[Mean, Transpose@rese], 0.001]],
    Join[{"SD of error means"},
     Round[Map[StandardDeviation, Transpose@rese], 0.001]],
    Join[{"means of SD of errors"}, Round[Map[Mean, Transpose@ress], 0.001]],
    Join[{"a"}, Round[Map[Mean, Transpose@resa], 0.001]],
    Join[{"GOF1"}, Round[Map[Mean, Transpose@resg], 0.001]],
    Join[{"GOFMC"}, Round[Map[Mean@*SelN, Transpose@resgofmc], 0.001]]
   } // TableForm];

Print[
  "****** Error Means and SDs of equation between X,Y, tested with W^{2a}"];
rese = {};
ress = {};
resa = {};
resg = {};
resf = {};
resgofmc = {};
kmax = 25; SelN[L_List] := Select[L, NumberQ];
Do[datR = isim[n];
  val0 = Join[Table[{X0[i], datR[[i, 1]]}, {i, 1, n}],
    Table[{Y0[i], datR[[i, 3]]}, {i, 1, n}]];
  r0 = slowSEM["W^{2a}", datR, {x1, x2, y1, y2}, {X0, Y0}, imodel0b, Quiet → True,
    InitValues → val0, LatentMean0 → False, userConstr → {c2 > 0, c4 > 0, a > 0}];
  r1 = slowSEM["W^{2a}", datR, {x1, x2, y1, y2}, {X0, Y0}, imodel1, Quiet → True,
    InitValues → val0, LatentMean0 → False, userConstr → {c2 > 0, c4 > 0, a > 0}];
  r2 = slowSEM["W^{2a}", datR, {x1, x2, y1, y2}, {X0, Y0}, imodel2, Quiet → True,
    InitValues → val0, LatentMean0 → False, userConstr → {c2 > 0, c4 > 0, a > 0}];
  AppendTo[resf, {r0@"Fmin", r1@"Fmin", r2@"Fmin"}];
  AppendTo[rese, {
    If[r0["ConvergenceOK"], Last@r0["EEmeans"], Null],
    If[r1["ConvergenceOK"], Last@r1["EEmeans"], Null],
    If[r2["ConvergenceOK"], Last@r2["EEmeans"], Null]}];
  AppendTo[ress, {
    If[r0["ConvergenceOK"], Sqrt@Last@r0["EVars"], Null],
    If[r1["ConvergenceOK"], Sqrt@Last@r1["EVars"], Null],
    If[r2["ConvergenceOK"], Sqrt@Last@r2["EVars"], Null]}];
  AppendTo[resa, {If[r0["ConvergenceOK"], a /. r0["sol"], Null],
    If[r1["ConvergenceOK"], a /. r1["sol"], Null],
    If[r2["ConvergenceOK"], a /. r2["sol"], Null]}];
  AppendTo[resgofmc, {If[r0["ConvergenceOK"], First@r0["GOFMC"], Null],
    If[r1["ConvergenceOK"], First@r1["GOFMC"], Null],
    If[r2["ConvergenceOK"], First@r2["GOFMC"], Null]}];
  AppendTo[resg, {If[r0["ConvergenceOK"], r0["GOF1"], Null],
    If[r1["ConvergenceOK"], r1["GOF1"], Null],
```

```
    If[r2["ConvergenceOK"], r2["GOF1"], Null]}],
  {k, kmax}];
Print[{{"Model:", "Corel", "Impl", "false Impl"},
    Join[{"Fmin"}, Round[Map[Mean, Transpose@resf], 0.001]],
    Join[{"M"}, Round[Map[Mean@*SelN, Transpose@rese], 0.001]],
    Join[{"SD of error means"},
     Round[Map[StandardDeviation@*SelN, Transpose@rese], 0.001]],
    Join[{"means of SD of errors"},
     Round[Map[Mean@*SelN, Transpose@ress], 0.001]],
    Join[{"a"}, Round[Map[Mean@*SelN, Transpose@resa], 0.001]],
    Join[{"GOF1"}, Round[Map[Mean@*SelN, Transpose@resg], 0.001]],
    Join[{"GOFMC"}, Round[Map[Mean@*SelN, Transpose@resgofmc], 0.001]]} //
   TableForm];

Print[
  "****** Error Means and SDs of equation between X,Y, tested with W^{2a0}"];
rese = {};
ress = {};
resa = {};
resg = {};
resf = {};
resgofmc = {};
kmax = 25; SelN[L_List] := Select[L, NumberQ];
Do[datR = isim[n];
  val0 = Join[Table[{X0[i], datR[[i, 1]]}, {i, 1, n}],
    Table[{Y0[i], datR[[i, 3]]}, {i, 1, n}]];
  r0 = slowSEM["W^{2a0}", datR, {x1, x2, y1, y2}, {X0, Y0}, imodel0b, Quiet → True,
    InitValues → val0, LatentMean0 → False, userConstr → {c2 > 0, c4 > 0, a > 0}];
  r1 = slowSEM["W^{2a0}", datR, {x1, x2, y1, y2}, {X0, Y0}, imodel1, Quiet → True,
    InitValues → val0, LatentMean0 → False, userConstr → {c2 > 0, c4 > 0, a > 0}];
  r2 = slowSEM["W^{2a0}", datR, {x1, x2, y1, y2}, {X0, Y0}, imodel2, Quiet → True,
    InitValues → val0, LatentMean0 → False, userConstr → {c2 > 0, c4 > 0, a > 0}];
  AppendTo[resf, {r0@"Fmin", r1@"Fmin", r2@"Fmin"}];
  AppendTo[rese, {
    If[r0["ConvergenceOK"], Last@r0["EEmeans"], Null],
    If[r1["ConvergenceOK"], Last@r1["EEmeans"], Null],
    If[r2["ConvergenceOK"], Last@r2["EEmeans"], Null]}];
  AppendTo[ress, {
    If[r0["ConvergenceOK"], Sqrt@Last@r0["EVars"], Null],
    If[r1["ConvergenceOK"], Sqrt@Last@r1["EVars"], Null],
    If[r2["ConvergenceOK"], Sqrt@Last@r2["EVars"], Null]}];
  AppendTo[resa, {If[r0["ConvergenceOK"], a /. r0["sol"], Null],
    If[r1["ConvergenceOK"], a /. r1["sol"], Null],
    If[r2["ConvergenceOK"], a /. r2["sol"], Null]}];
  AppendTo[resgofmc, {If[r0["ConvergenceOK"], First@r0["GOFMC"], Null],
    If[r1["ConvergenceOK"], First@r1["GOFMC"], Null],
    If[r2["ConvergenceOK"], First@r2["GOFMC"], Null]}];
```

```
  AppendTo[resg, {If[r0["ConvergenceOK"], r0["GOF1"], Null],
    If[r1["ConvergenceOK"], r1["GOF1"], Null],
    If[r2["ConvergenceOK"], r2["GOF1"], Null]}],
  {k, kmax}];
Print[{{"Model:", "Corel", "Impl", "false Impl"},
    Join[{"Fmin"}, Round[Map[Mean, Transpose@resf], 0.001]],
    Join[{"M"}, Round[Map[Mean@*SelN, Transpose@rese], 0.001]],
    Join[{"SD of error means"},
     Round[Map[StandardDeviation@*SelN, Transpose@rese], 0.001]],
    Join[{"means of SD of errors"},
     Round[Map[Mean@*SelN, Transpose@ress], 0.001]],
    Join[{"a"}, Round[Map[Mean@*SelN, Transpose@resa], 0.001]],
    Join[{"GOF1"}, Round[Map[Mean@*SelN, Transpose@resg], 0.001]],
    Join[{"GOFMC"}, Round[Map[Mean@*SelN, Transpose@resgofmc], 0.001]]} //
   TableForm];
```

***** Error Means and SDs of equation between X,Y, tested with W1

	Corel	Impl	false Impl
Fmin	0.303	0.16	0.283
M	0.	0.008	0.247
SD of error means	0.	0.003	0.028
means of SD of errors	0.394	0.095	0.263
a	0.379	0.796	0.32
GOF1	0.27	0.214	0.248
GOFMC	0.671	0.491	0.65

****** Error Means and SDs of equation between X,Y, tested with W^{2a}

Model:	Corel	Impl	false Impl
Fmin	0.478	0.219	0.529
M	0.	0.	0.
SD of error means	0.	0.	0.
means of SD of errors	0.457	0.02	0.461
a	0.787	0.881	1.406
GOF1	10.304	4.079	7.44
GOFMC	0.447	0.33	0.408

****** Error Means and SDs of equation between X,Y, tested with W^{2a0}

Model:	Corel	Impl	false Impl
Fmin	0.289	0.186	0.472
M	0.	0.	0.
SD of error means	0.	0.	0.
means of SD of errors	0.362	0.018	0.39
a	0.405	0.938	1.521
GOF1	25.792	17.258	20.376
GOFMC	0.647	0.457	0.547

In[171]:=

```
ST = SemTest[Nsimu, {100, 300}, isim, fastM, slowM,
   Iparas, Itrue, {{x1, x2, y1, y2}, {X0, Y0}, imodel1},
   userConstr → {c2 > 0, c4 > 0, a > 0}, LatentMean0 → False, LatInit → {1, 3},
   ExportData → "~/Dropbox/Statistik/sem_studie/impl/dat/"];
Print@ST["ltx"]
```

```
------------------------------------------------------ 100
```

Method	{c2, c4, a}	std	#fails	avg time	avg GOF	SD GOF
W1	{0.644, 0.541, 0.783}	{0.053, 0.077, 0.061}	0	0.5	0.496	0.015
Wn	{0.64, 0.553, 0.749}	{0.053, 0.077, 0.059}	0	0.6	0.537	0.018
Ww(Wn)	{0.604, 0.442, 0.78}	{0.055, 0.075, 0.064}	0	1.1	0.49	0.022
W^{2a}	{0.701, 0.52, 0.885}	{0.06, 0.069, 0.076}	0	7.2	0.334	0.011
W^{2a0}	{0.698, 0.406, 0.951}	{0.08, 0.063, 0.101}	1	7.4	0.463	0.015

```
------------------------------------------------------ 300
```

Method	{c2, c4, a}	std	#fails	avg time	avg GOF	SD GOF
W1	{0.641, 0.535, 0.78}	{0.043, 0.067, 0.052}	0	1.6	0.494	0.013
Wn	{0.636, 0.547, 0.744}	{0.043, 0.067, 0.05}	0	1.6	0.537	0.015
Ww(Wn)	{0.6, 0.436, 0.776}	{0.045, 0.064, 0.053}	0	3.2	0.489	0.019
W^{2a}	{0.695, 0.511, 0.876}	{0.05, 0.058, 0.065}	2	21.5	0.332	0.01
W^{2a0}	{0.692, 0.4, 0.944}	{0.067, 0.057, 0.093}	1	21.9	0.46	0.014

```
\begin{table}
         \caption{Titel (100 simulations for each $n$) }
        {\footnotesize\begin{tabular}{ccccccc}\hline
$n$ & var   & $W_1$  & $W_n$  & $Ww(W_n)$  & $W^{2a}$  & $W^{2a0}$ \\
100 & $c_2$ & -0.06 & -0.06 & -0.1 & 0. & 0.\\
 & 0.7 & (0.05) & (0.05) & (0.06) & (0.06) & (0.08)\\
 & RMSE & 0.08 & 0.08 & 0.11 & 0.06 & 0.08\\
 & $c_4$ & 0.14 & 0.15 & 0.04 & 0.12 & 0.01\\
 & 0.4 & (0.08) & (0.08) & (0.07) & (0.07) & (0.06)\\
 & RMSE & 0.16 & 0.17 & 0.09 & 0.14 & 0.06\\
 & $a$ & -0.12 & -0.15 & -0.12 & -0.01 & 0.05\\
 & 0.9 & (0.06) & (0.06) & (0.06) & (0.08) & (0.1)\\
 & RMSE & 0.13 & 0.16 & 0.14 & 0.08 & 0.11\\
 & GOF  & 0.496 & 0.537 & 0.49 & 0.334 & 0.463\\
 & time  & 0.5 & 0.6 & 1.1 & 7.2 & 7.4\\
100 & fails  & 0 & 0 & 0 & 0 & 1\\
\hline
300 & $c_2$ & -0.06 & -0.06 & -0.1 & 0. & -0.01\\
 & 0.7 & (0.04) & (0.04) & (0.04) & (0.05) & (0.07)\\
 & RMSE & 0.07 & 0.08 & 0.11 & 0.05 & 0.07\\
 & $c_4$ & 0.13 & 0.15 & 0.04 & 0.11 & 0.\\
 & 0.4 & (0.07) & (0.07) & (0.06) & (0.06) & (0.06)\\
 & RMSE & 0.15 & 0.16 & 0.07 & 0.13 & 0.06\\
 & $a$ & -0.12 & -0.16 & -0.12 & -0.02 & 0.04\\
 & 0.9 & (0.05) & (0.05) & (0.05) & (0.06) & (0.09)\\
 & RMSE & 0.13 & 0.16 & 0.14 & 0.07 & 0.1\\
 & GOF  & 0.494 & 0.537 & 0.489 & 0.332 & 0.46\\
 & time  & 1.6 & 1.6 & 3.2 & 21.5 & 21.9\\
300 & fails  & 0 & 0 & 0 & 2 & 1\\
\hline
\hline
    \end{tabular}}
    \label{tab:1}
\end{table}
```

```
(***************************** Competence
 model ************************************)
```

In[173]:=

```
modelK = {
```

```
    {item1 == 0 * u1 + c11 * L1 + c13 * L1 * K1, e1},
    {item2 == 0 * u2 + c22 * K1 + c23 * L1 * K1, e2},
    {item3 == 0 * u3 + c3 * L1, e3},
    {item4 == 0 * u4 + c4 * K1, e4},
    {item5 == 0 * u5 + c5 * L1, e5},
    {item6 == 0 * u6 + c6 * K1, e6}};
paras = {c11, c13, c22, c23, c3, c4, c5, c6};
truesol = {c11 → 0.4, c13 → 0.6 ,
   c22 → 0.3 , c23 → 0.7, c3 → 0.9, c4 → 0.8, c5 → 0.7, c6 → 0.7};
Ktrue = paras /. truesol;

KoSimu[n_] :=
  Module[{L1K1true, K1true, L1true, ItrueN, Item1, Item2, Item3, Item4, Item5,
    Item6, dataK, c11trueN, c12trueN, c13trueN, c21trueN, c22trueN, c23trueN,
    c3trueN, c4trueN, c5trueN, c6trueN},
   L1K1true = RandomVariate[UniformDistribution[{0, 0.1}], n];
   K1true = L1K1true + RandomVariate[UniformDistribution[{0, 0.9}], n];
   L1true = L1K1true + RandomVariate[UniformDistribution[{0, 0.8}], n];
   Item1 = ((0. + c11 * L1true + c13 * L1true * K1true) /. truesol) +
     RandomVariate[NormalDistribution[0, 0.2], n];
   Item2 = ((0.0 + c22 * K1true + c23 * L1true * K1true) /. truesol) +
     RandomVariate[NormalDistribution[0, 0.1], n];
   Item3 = 0.0 + c3 * L1true + RandomVariate[NormalDistribution[0, 0.2], n];
   Item4 = 0.0 + c4 * K1true + RandomVariate[NormalDistribution[0, 0.1], n];
   Item5 =
    (0.0 + c5 * L1true + RandomVariate[NormalDistribution[0, 0.2], n]) /. truesol;
   Item6 =
    (0.0 + c6 * K1true + RandomVariate[NormalDistribution[0, 0.1], n]) /. truesol;
   dataK = Transpose@{Item1, Item2, Item3, Item4, Item5, Item6} /. truesol;
   c11trueN =
    (c11 /. truesol) * StandardDeviation[L1true] / StandardDeviation[Item1];
   c13trueN = (c13 /. truesol) *
     StandardDeviation[L1true * K1true] / StandardDeviation[Item1];
   c22trueN =
    (c22 /. truesol) * StandardDeviation[K1true] / StandardDeviation[Item2];
   c23trueN = (c23 /. truesol) *
     StandardDeviation[L1true * K1true] / StandardDeviation[Item2];
   c3trueN =
    (c3 /. truesol) * StandardDeviation[L1true] / StandardDeviation[Item3];
   c4trueN =
    (c4 /. truesol) * StandardDeviation[K1true] / StandardDeviation[Item4];
   c5trueN =
    (c5 /. truesol) * StandardDeviation[L1true] / StandardDeviation[Item5];
   c6trueN =
    (c6 /. truesol) * StandardDeviation[K1true] / StandardDeviation[Item6];
   ItrueN = {c11trueN, c13trueN,
     c22trueN, c23trueN, c3trueN, c4trueN, c5trueN, c6trueN};
```

```
{dataK, ItrueN, L1true, K1true} /. truesol];

n = 300; {datR, ItrueN, L1true, K1true} = KoSimu[n];
Print["true sim"];
NormedParaTrue = KoSimu[100 000][[2]];
Print[{paras, Ktrue, Round[NormedParaTrue, 0.01]} // TableForm];
Print[MyRound[{"L mean", Map[Mean, {L1true, K1true}],
     "L Std", Map[StandardDeviation, {L1true, K1true}]}, 0.01]];

Print["fast"];
res = fastSEM[datR, {item1, item2, item3, item4, item5, item6},
    {L1, K1}, modelK, LatentMean0 → False, calcDataFit → 1,
    userConstr → Map[# > 0 &, paras],
    userLatentConstr → {L1 > 0, K1 > 0, K1 < 1, L1 < 1}];
Print[
  MyRound[{paras, paras /. res["sol"], paras /. res["NormedParas"]}, 0.01] //
    TableForm];
Print[MyRound[
    {"L mean", Map[Mean, res["LLs"]], "L Std", Map[StandardDeviation, res["LLs"]],
     "GOF1,DF", res["GOF1"], res["dataFit"]}, 0.01]];

If[False, Print["W^2a0"];
  res2 = slowSEM["W^{2a0}", datR,
    {item1, item2, item3, item4, item5, item6}, {L1, K1}, modelK,
    calcDataFit → 1, userConstr → Join[Map[# > 0 &, paras], Map[# < 10 &, paras]],
    userLatentConstr → {L1 > 0, K1 > 0, K1 < 1, L1 < 1}, LatentMean0 → False];
  Print[MyRound[{paras, paras /. res2["sol"],
      paras /. res2["NormedParas"]}, 0.01] // TableForm];
  Print[
   MyRound[{"L mean", Map[Mean, res2["LLs"]], "L Std", Map[StandardDeviation,
      res2["LLs"]], "GOF1,DF", res2["GOF1"], res2["dataFit"]}, 0.01]];
 ];
If[True, Print["W^2a"];
  res2 = slowSEM["W^{2a}", datR,
    {item1, item2, item3, item4, item5, item6}, {L1, K1}, modelK,
    calcDataFit → 1, userConstr → Join[Map[# > 0 &, paras], Map[# < 10 &, paras]],
    userLatentConstr → {L1 > 0, K1 > 0, K1 < 1, L1 < 1}, LatentMean0 → False];
  Print[MyRound[{paras, paras /. res2["sol"],
      paras /. res2["NormedParas"]}, 0.01] // TableForm];
  Print[
   MyRound[{"L mean", Map[Mean, res2["LLs"]], "L Std", Map[StandardDeviation,
      res2["LLs"]], "GOF1,DF", res2["GOF1"], res2["dataFit"]}, 0.01]];
 ];
If[True, n = 100;
  ST = SemTest[10 + 0 * Nsimu, {n}, First[KoSimu[#]] &, fastM, slowM, paras, Ktrue,
    {{item1, item2, item3, item4, item5, item6}, {L1, K1}, modelK},
    printSummary → True, normed → True, normedTrue → KoSimu[100 000][[2]],
```

```
    userConstr → Join[Map[# > 0 &, paras], Map[# < 10 &, paras]],
    LatentMean0 → False,
    userLatentConstr → {L1 > 0, K1 > 0, K1 < 1, L1 < 1}, LatInit → {3, 4}];
  Print@ST["ltx"] ];
If[False, n = 300;
 ST = SemTest[Nsimu, {n}, First[KoSimu[#]] &,  fastM, slowM, paras, Ktrue,
   {{item1, item2, item3, item4, item5, item6}, {L1, K1}, modelK},
   printSummary → True, normed → True, normedTrue → KoSimu[100 000][[2]],
   userConstr → Join[Map[# > 0 &, paras], Map[# < 10 &, paras]],
   LatentMean0 → False,
   userLatentConstr → {L1 > 0, K1 > 0, K1 < 1, L1 < 1}, LatInit → {3, 4}];
 Print@ST["ltx"] ]
```

```
true sim
```

```
c11     c13     c22     c23     c3      c4      c5      c6
0.4     0.6     0.3     0.7     0.9     0.8     0.7     0.7
0.34    0.39    0.37    0.59    0.72    0.9     0.63    0.88
```

```
{L mean, {0.44, 0.47}, L Std, {0.24, 0.26}}
```

```
fast
```

```
c11     c13     c22     c23     c3      c4      c5      c6
0.25    1.46    0.39    0.99    1.15    0.96    0.89    0.88
0.21    0.66    0.4     0.54    0.83    0.93    0.77    0.92
```

```
{L mean, {0.36, 0.38}, L Std, {0.22, 0.23}, GOF1,DF, 0.16, 0.2}
```

```
W^2a
```

```
c11     c13     c22     c23     c3      c4      c5      c6
0.38    2.3     0.37    1.89    1.54    1.14    1.22    1.04
0.26    0.74    0.33    0.74    0.91    0.96    0.87    0.95
```

```
{L mean, {0.27, 0.33}, L Std, {0.18, 0.2}, GOF1,DF, 6.56, 0.25}
```

```
------------------------------------------------------ 100
```

```
⎛ Method            {c11, c13, c22, c23, c3, c4, c5, c6}
  W1     {0.348, 0.517, 0.478, 0.509, 0.834, 0.928, 0.74, 0.903}  {0.178, 0.174, 0.058, 0.(
  Wn     {0.439, 0.364, 0.523, 0.448, 0.862, 0.947, 0.762, 0.922}  {0.124, 0.103, 0.047, 0.
 Ww(Wn)  {0.422, 0.403, 0.474, 0.526, 0.862, 0.946, 0.732, 0.915}  {0.128, 0.11, 0.048, 0.(
 W^{2a}  {0.394, 0.565, 0.429, 0.679, 0.906, 0.966, 0.809, 0.94}  {0.208, 0.212, 0.091, 0.(
⎝W^{2a0} {0.412, 0.413, 0.407, 0.618, 0.806, 0.938, 0.72, 0.913}  {0.152, 0.131, 0.074, 0.(
```

```
\begin{table}
         \caption{Titel (10 simulations for each $n$) }
        {\footnotesize\begin{tabular}{ccccccc}\hline
$n$ & var   & $W_1$  & $W_n$  & $Ww(W_n)$  & $W^{2a}$  & $W^{2a0}$ \\
100 & $c11$ & 0. & 0.09 & 0.08 & 0.05 & 0.07\\
 & 0.4 & (0.18) & (0.12) & (0.13) & (0.21) & (0.15)\\
 & RMSE & 0.18 & 0.12 & 0.12 & 0.2 & 0.14\\
 & $c13$ & 0.12 & -0.03 & 0.01 & 0.17 & 0.02\\
 & 0.6 & (0.17) & (0.1) & (0.11) & (0.21) & (0.13)\\
 & RMSE & 0.18 & 0.26 & 0.22 & 0.2 & 0.22\\
 & $c22$ & 0.11 & 0.15 & 0.1 & 0.06 & 0.04\\
 & 0.3 & (0.06) & (0.05) & (0.05) & (0.09) & (0.07)\\
 & RMSE & 0.19 & 0.23 & 0.18 & 0.16 & 0.13\\
 & $c23$ & -0.08 & -0.14 & -0.06 & 0.09 & 0.03\\
 & 0.7 & (0.07) & (0.06) & (0.06) & (0.08) & (0.08)\\
 & RMSE & 0.2 & 0.26 & 0.18 & 0.08 & 0.11\\
 & $c_3$ & 0.11 & 0.14 & 0.14 & 0.18 & 0.08\\
 & 0.9 & (0.04) & (0.03) & (0.03) & (0.05) & (0.06)\\
 & RMSE & 0.08 & 0.05 & 0.05 & 0.05 & 0.11\\
 & $c_4$ & 0.03 & 0.05 & 0.04 & 0.06 & 0.04\\
 & 0.8 & (0.02) & (0.02) & (0.02) & (0.02) & (0.03)\\
 & RMSE & 0.13 & 0.15 & 0.15 & 0.17 & 0.14\\
 & $c_5$ & 0.11 & 0.13 & 0.1 & 0.18 & 0.09\\
 & 0.7 & (0.04) & (0.04) & (0.04) & (0.06) & (0.05)\\
 & RMSE & 0.05 & 0.07 & 0.05 & 0.12 & 0.05\\
 & $c_6$ & 0.02 & 0.04 & 0.03 & 0.06 & 0.03\\
 & 0.7 & (0.03) & (0.02) & (0.02) & (0.03) & (0.03)\\
 & RMSE & 0.2 & 0.22 & 0.22 & 0.24 & 0.21\\
 & GOF  & 0.422 & 0.475 & 0.466 & 0.341 & 0.419\\
 & time  & 0.7 & 0.7 & 1.5 & 7.4 & 7.3\\
\hline
\hline
    \end{tabular}}
    \label{tab:1}
\end{table}
```

In[191]:=

```
If[True, n = 300;
 ST = SemTest[Nsimu, {n}, First[KoSimu[#]] &,  fastM, slowM, paras, Ktrue,
   {{item1, item2, item3, item4, item5, item6}, {L1, K1}, modelK},
   printSummary → True, normed → True, normedTrue → KoSimu[100 000][[2]],
   userConstr → Join[Map[# > 0 &, paras], Map[# < 10 &, paras]],
   LatentMean0 → False,
   userLatentConstr → {L1 > 0, K1 > 0, K1 < 1, L1 < 1}, LatInit → {3, 4}];
 Print@ST["ltx"] ]
```

```
---------------------------------------------------- 300
```

```
( Method            {c11, c13, c22, c23, c3, c4, c5, c6}
    W1      {0.3, 0.553, 0.45, 0.526, 0.838, 0.934, 0.735, 0.909}    {0.08, 0.082, 0.042, 0.(
    Wn     {0.391, 0.384, 0.496, 0.46, 0.866, 0.949, 0.76, 0.924}   {0.06, 0.064, 0.042, 0.0
  Ww(Wn)  {0.382, 0.417, 0.438, 0.55, 0.862, 0.949, 0.729, 0.917}   {0.061, 0.064, 0.044, 0
  W^{2a}  {0.352, 0.603, 0.392, 0.715, 0.923, 0.968, 0.811, 0.941} {0.098, 0.119, 0.074, 0.(
 W^{2a0}   {0.389, 0.42, 0.379, 0.634, 0.82, 0.941, 0.72, 0.916}    {0.064, 0.067, 0.054, 0.
```

```
\begin{table}
         \caption{Titel (100 simulations for each $n$) }
        {\footnotesize\begin{tabular}{ccccccc}\hline
$n$ & var   & $W_1$  & $W_n$  & $Ww(W_n)$  & $W^{2a}$  & $W^{2a0}$ \\
300 & $c11$ & -0.05 & 0.05 & 0.04 & 0.01 & 0.04\\
 & 0.4 & (0.08) & (0.06) & (0.06) & (0.1) & (0.06)\\
 & RMSE & 0.13 & 0.06 & 0.06 & 0.11 & 0.06\\
 & $c13$ & 0.16 & -0.01 & 0.02 & 0.21 & 0.03\\
 & 0.6 & (0.08) & (0.06) & (0.06) & (0.12) & (0.07)\\
 & RMSE & 0.09 & 0.23 & 0.19 & 0.12 & 0.19\\
 & $c22$ & 0.08 & 0.12 & 0.07 & 0.02 & 0.01\\
 & 0.3 & (0.04) & (0.04) & (0.04) & (0.07) & (0.05)\\
 & RMSE & 0.16 & 0.2 & 0.15 & 0.12 & 0.1\\
 & $c23$ & -0.06 & -0.13 & -0.04 & 0.13 & 0.05\\
 & 0.7 & (0.04) & (0.04) & (0.05) & (0.07) & (0.05)\\
 & RMSE & 0.18 & 0.24 & 0.16 & 0.07 & 0.08\\
 & $c_3$ & 0.11 & 0.14 & 0.14 & 0.2 & 0.1\\
 & 0.9 & (0.02) & (0.02) & (0.02) & (0.03) & (0.03)\\
 & RMSE & 0.07 & 0.04 & 0.04 & 0.03 & 0.09\\
 & $c_4$ & 0.03 & 0.05 & 0.04 & 0.06 & 0.04\\
 & 0.8 & (0.02) & (0.01) & (0.01) & (0.02) & (0.02)\\
 & RMSE & 0.13 & 0.15 & 0.15 & 0.17 & 0.14\\
 & $c_5$ & 0.1 & 0.13 & 0.1 & 0.18 & 0.09\\
 & 0.7 & (0.03) & (0.02) & (0.03) & (0.04) & (0.03)\\
 & RMSE & 0.04 & 0.06 & 0.04 & 0.12 & 0.04\\
 & $c_6$ & 0.03 & 0.05 & 0.04 & 0.06 & 0.04\\
 & 0.7 & (0.02) & (0.02) & (0.02) & (0.02) & (0.02)\\
 & RMSE & 0.21 & 0.22 & 0.22 & 0.24 & 0.22\\
 & GOF  & 0.403 & 0.46 & 0.451 & 0.339 & 0.408\\
 & time  & 3.1 & 3.2 & 6.4 & 27.2 & 26.7\\
\hline
\hline
    \end{tabular}}
    \label{tab:1}
\end{table}
```

In[146]:=

```
(******** Nonlinear Mediator model ***********)
Mlat = {X, Y, Z}; Mobs = {x1, x2, y1, y2, z1, z2};
ModEqsNL = {{Y == k1 * X + u1, e1}, {Z == k2 * Y + k3 * X^3 + u2, e2},
    {x1 == 1 * X(*+t1*), ee1}, {x2 == a2 * X + t2, ee2},
    {y1 == 1 * Y(*+t3*), ee3}, {y2 == b2 * Y + t4, ee4},
    {z1 == 1 * Z(*+t5*), ee5}, {z2 == c2 * Z + t6, ee6} };
SimDataModNL[n_, norm_, ks_, sigY_,
    sigZ_, sigX1_, sigX2_, sigY1_, sigY2_, sigZ1_, sigZ2_] :=
  Module[{i, XX, YY, ZZ, X1, X2, Y1, Y2, Z1, Z2},
    EVR = If[norm, NVR, UVR];
    XX = If[norm, NVR[1, n], UVR[1, n]];
    YY = ks[[1]] * XX + EVR[sigY, n];
    ZZ = ks[[2]] * YY + ks[[3]] * XX^3 + EVR[sigZ, n];
    X1 = 1.0 * XX + EVR[sigX1, n]; X2 = 1.2 * XX + EVR[sigX2, n];
    Y1 = 1.0 * YY + EVR[sigY1, n]; Y2 = 0.7 * YY + EVR[sigY2, n];
    Z1 = 1.0 * ZZ + EVR[sigZ1, n]; Z2 = 0.9 * ZZ + EVR[sigZ2, n];
    Transpose[{X1, X2, Y1, Y2, Z1, Z2}]];
ktrue = {0.5, 0.8, 0.4};
ST = SemTest[Nsimu, {300},
  SimDataModNL[#, True, ktrue, 0.4, 0.3, 0.3, 0.3, 0.2, 0.2, 0.4, 0.2 ] &,
  fastM, slowM, {k1, k2, k3}, ktrue, {Mobs, Mlat, ModEqsNL}, LatentMean0 → False,
  printSummary → True, LatInit → {1, 3, 5}]; Print@ST["ltx"]
```

```
------------------------------------------------------ 300
```

Method	{k1, k2, k3}	std	#fails	avg time	avg GOF	SD GOF
W1	{0.442, 1.288, 0.333}	{0.032, 0.137, 0.029}	0	13.9	0.435	0.015
Wn	{0.46, 0.817, 0.356}	{0.088, 0.182, 0.068}	0	3.3	0.419	0.123
Ww(Wn)	{0.497, 0.772, 0.4}	{0.03, 0.071, 0.031}	0	6.9	0.401	0.009
W^{2a}	{0.619, 0.586, 0.46}	{0.031, 0.083, 0.035}	1	86.4	0.233	0.005
W^{2a0}	{0.481, 0.814, 0.382}	{0.032, 0.085, 0.031}	1	71.4	0.401	0.012

```
\begin{table}
         \caption{Titel (100 simulations for each $n$) }
        {\footnotesize\begin{tabular}{ccccccc}\hline
$n$ & var   & $W_1$  & $W_n$  & $Ww(W_n)$  & $W^{2a}$  & $W^{2a0}$ \\
300 & $k_1$ & -0.06 & -0.04 & 0. & 0.12 & -0.02\\
 & 0.5 & (0.03) & (0.09) & (0.03) & (0.03) & (0.03)\\
 & RMSE & 0.07 & 0.1 & 0.03 & 0.12 & 0.04\\
 & $k_2$ & 0.49 & 0.02 & -0.03 & -0.21 & 0.01\\
 & 0.8 & (0.14) & (0.18) & (0.07) & (0.08) & (0.09)\\
 & RMSE & 0.51 & 0.18 & 0.08 & 0.23 & 0.09\\
 & $k_3$ & -0.07 & -0.04 & 0. & 0.06 & -0.02\\
 & 0.4 & (0.03) & (0.07) & (0.03) & (0.04) & (0.03)\\
 & RMSE & 0.07 & 0.08 & 0.03 & 0.07 & 0.04\\
 & GOF  & 0.435 & 0.419 & 0.401 & 0.233 & 0.401\\
 & time  & 13.9 & 3.3 & 6.9 & 86.4 & 71.4\\
300 & fails  & 0 & 0 & 0 & 1 & 1\\
\hline
\hline
    \end{tabular}}
    \label{tab:1}
\end{table}
```