FeliX –Tutorial

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FeliX is a new kind oft program that combines dynamic geometry and computer algebra in a novel way. The user has full access to the geometric and the algebraic representation. Equations and inequalities are used to model geometric situations.

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Copyright, Installation and Start

Copyright

FeliX if free for private use and for use in public schools. However, all rights are reserved.
FeliX uses various other software components, not of all are free.

- The computer algebra system MuPAD (www.sciface.com)
- The optimization library DONLP2 (Peter Spellucci).
- The Java compiler Janino (Arno Unkrig http://janino.codehaus.org).
- The functions parser and byte code compiler „Function parser v2.7 by Warp“.

System requirements

Hardware: PC with at least 256MB, fast processor
Software: MuPAD 4.0.1 or MuPAD 4.0.2

1. Installation of MuPAD
2. Installation of the MuPAD-Java-Interface (www.ph-heidelberg.de/wp/oldenburg/felix)
3. Installation of FeliX

- Download the file felix???.zip from www.ph-heidelberg.de/wp/oldenburg/felix and extract it to an arbitrary directory.
- Start (with admin rights) the installation program `FelixInstaller.exe`.
- Choose the right MuPAD version and the installation directory and press the install button.

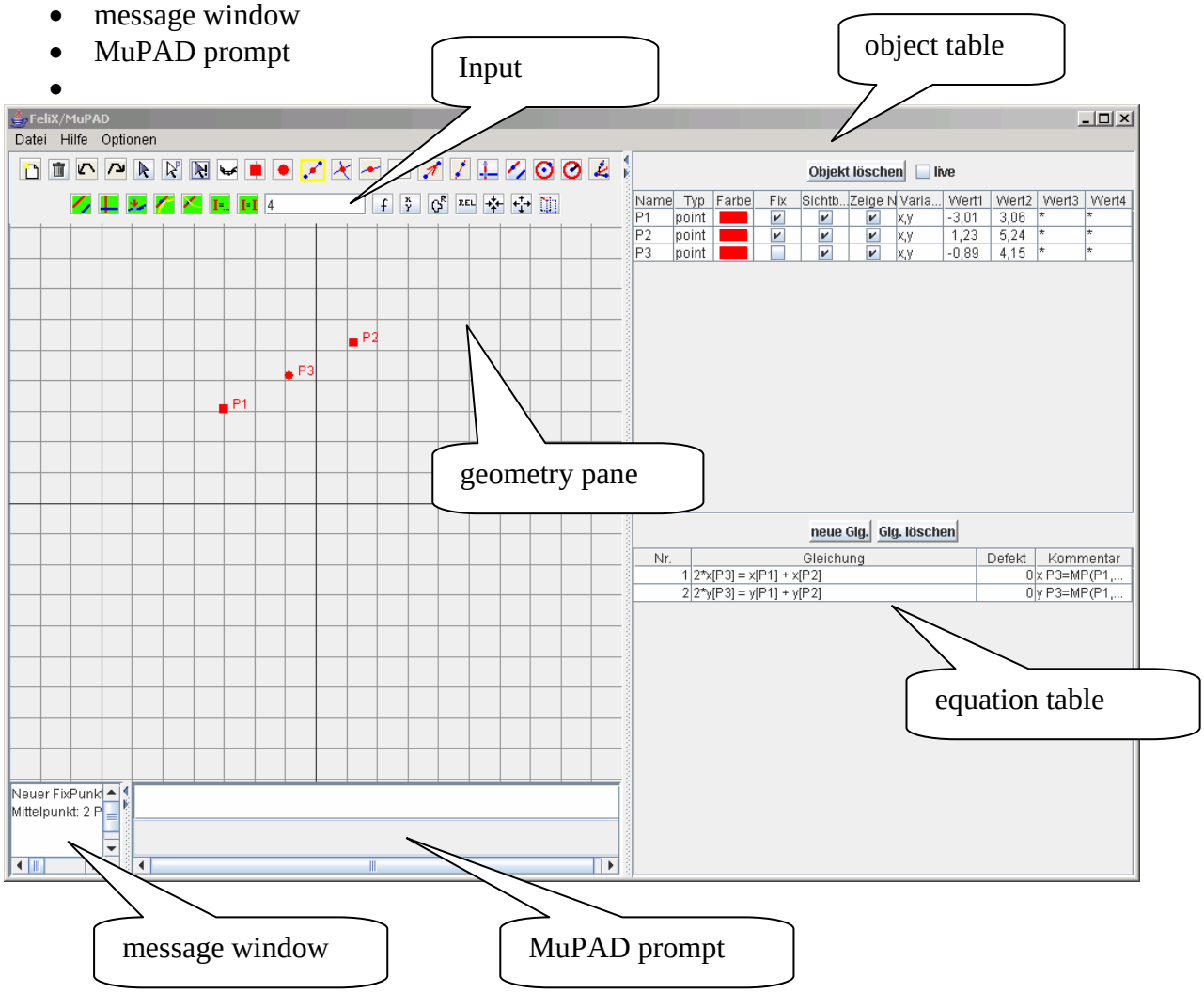
To start up FeliX the file `felix001.mu` from the directory `packages\FeliX must be read into MuPAD. This can be done with the` MuPAD notebook `start_felix`.

Change of language: Enter the options (Optionen) menu and select language (Sprache) and open the FeliX language file in the directory packages\felix\lang.

First steps

The FeliX window is composed of the following components:

- Menu bar
- buttons
- geometry pane that sows a region of the Euclidean plane
- object table
- equation table
- message window
- MuPAD prompt
-



Example 1: Midpoint of two points

In this example we construct two points and their midpoint.

Click to select point creation mode. Now click at two places in the geometry window. Two pints P1 and P2 will be created. They appear as well in the object table.

Click to select midpoint creation mode. Now click P1 and P2. The midpoint will be created and the relevant equations appear in the equation table.

Two drag the construction click . Thereafter one can click P1 or P2 and move it by dragging. The coordinates of the points in the object table will be updated when the mouse button is released.

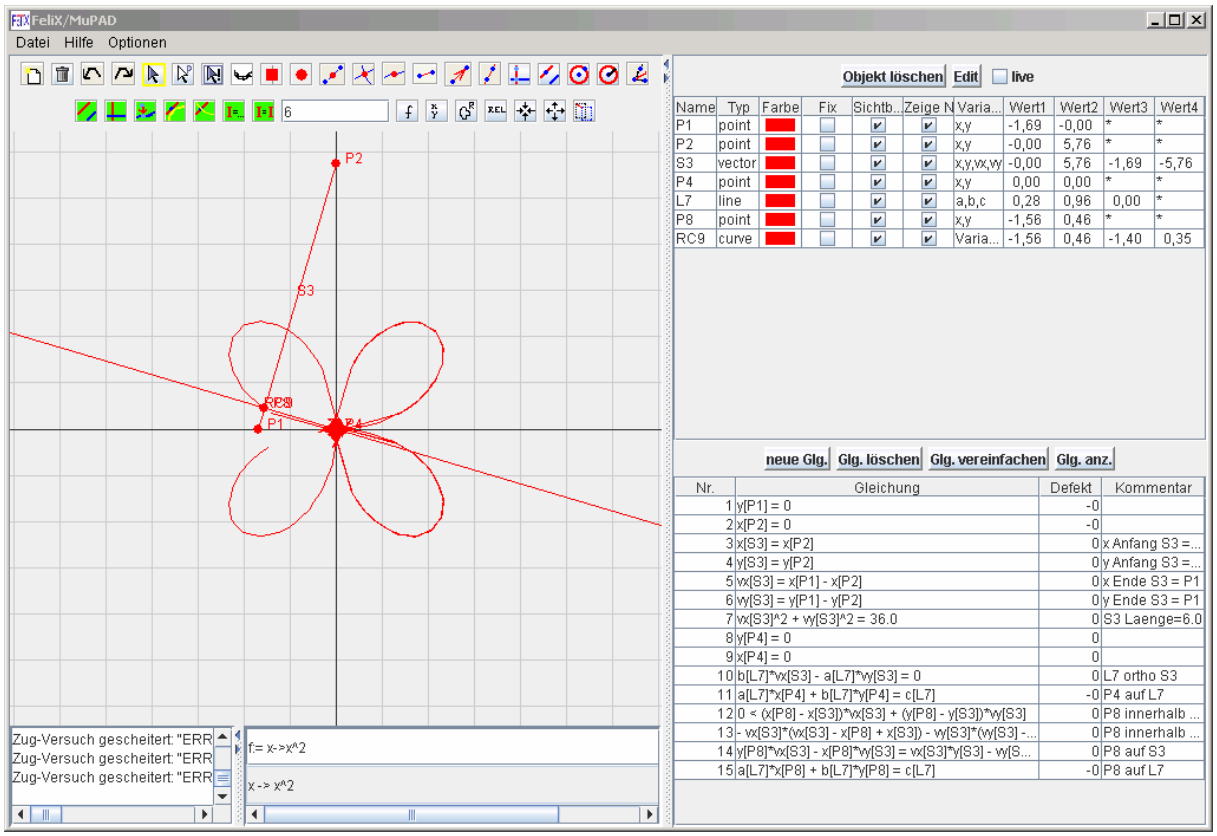
All objects in the object table have a checkbox “fix”. This can be changed at any time. Fixed points will be drawn as small squares, other points as small filled circles. Whether an object is fix or not determines the way it behaves during dragging. The rule is:
Fixed objects change their coordinates only when dragged directly, bur never as a reaction to dragging of other objects. All movements respect the equations in the equation browser.
If in the midpoint construction P1 and P are fix, P3 cannot be moved because with fixed values for P1 and P2 there is no freedom left for P3. But if you uncheck the checkbox of P2, P3 gets free, because now P2 can be adjusted. You may even unfix P1 to get a fully flexible construction of fix P3.

The coordinates of objects may also be changed by clicking and editing the cell in the object table.
Moreover, the cells for the name of the objects and its colour are editable as well.

The equation table has two entries:
```
2*x[P3]=x[P1]+x[P2]
2*y[P3]=y[P1]+y[P2]
```
They are editable as well. E.g. one may change them to:
```
2*x[P3]=x[P1]+x[P2]
2*y[P3]=y[P1]+y[P2]
```
Observe the effect of this change.
Now, make sure that P1 is not fix, press the “New Eq.” button and edit the newly created trivial equation 0=0 to:
```
x[P1]=0
```
Again, observe the impact.

Example 2: A curve

Create one point on the x- and one on the y-axis by clicking (mode of creation of points on objects) and click one on the x- and once on the y-axis. Observe that equations are created.
Create a segment between these points by clicking followed by P1 and P2.
The length of this segment shall be set to 6. To do this enter 6 in the input field, click and the segment.
Now create a point at the origin using . Check that two equations are created in this step.
Create the perpendicular line by clicking , the point at the origin and the segment.
To create the intersection point click the segment and the line.
The created intersection point can be moved with the mouse, but not freely. It is restrict to a curve which we call in this context its relation curve. This curve can be calculated by clicking REL and the point.:



Selecting the curve in the object table (or by right clicking it in the geometry pane) and pressing the object table's edit button one can read off the equation and edit it as well.

Exercise: Delete all objects other than P1, P2 and the segment. Create the midpoint of P1 and P2 and record its curve.

Variables and expressions in FeliX

FeliX processes all information algebraically. During dragging it uses only the information given in the equation table and whether the objects are fixed or not.

Variables

The position of geometric objects is determined by the values of their coordinates. They are denoted using square brackets. For example, the x- and y-coordinates of point P1 are denoted as x[P1] and y[P1].

Object class	Variables	Meaning
point P	`x[P], y[P]`	Cartesian coordinates
circle C	`x[C], y[C], r[C]`	midpoint and radius
line L	`a[L], b[L], c[L]`	coefficients in *ax+by=c*
vector V	`x[V], y[V], xv[V], yv[V]`	coordinates of start point and vector components. The endpoint is (x[V]+xv[V],y[V]+yv[V])

Expressions and Equations
Expressions and equations are valid if they are composed of:

- The variables of already defined objects
- `+, *, -, /, ^, =, <, <=,>, >=`
- The functions `sin, cos, tan, arcsin, arccos, arctan, sinh, cosh, tanh, ln, log, exp, abs, sign, sqrt`
- The constants `E, PI` and other MuPAD-expressions that can be evaluated by the MuPAD function `float` to a floating point number.
- User functions and variables defined in MuPAD if their application yields results which are valid according to the above rules. Examples of such predefined user functions are: `dist, len, angle`.

All expressions must be built according to MuPAD syntax rules, i.e.

- The multiplication sign may not be omitted
- decimal numbers are entered using a decimal point
- no spaces are allowed within names of objects

Examples:
```
x[P1]=x[P2]+1
sqrt(xv[V2]^2+y v[V2]^2)=12.2
abs(x[P1])<y[P1]
dist(P1,P2)=len(S3)
```

Some Functions:

dist(A,B)
Distance between A and B. These may be points or lines.

dist2(A,B)
square of dist

len(A)
Length of an object. A may be a segment, vector or circle (then length is the circumference)

angle(g,h)
The angle in radians between g and h. The objects may be lines, segments or vectors.

Objects and the object table

While traditional geometry systems have many different types of points (free, midpoint, intersection….) there is only one type of point in FeliX. A point on an object is just a point together with an equation that constrains it to be on the points. There are only the following kinds of objects:

- point
- circle
- line
- segment / vector (they differ only in the way they are drawn)
- function graphs (curve)

- parametric curve (curve)
- implicit curve (curve)

The columns in the object table are:
Name: Name of the object, ma be edited.
Type: point, line, circle, vector, curve.
Farbe(color): may be edited
Fix:Is the object fix? (may be changed)
Sichtbar(visible) : (may be changed)
Zeige Namen (show name): (may be changed)
Variablen: List os variables of the object. (may not be edited)
Wert1, Wert2 ,...: Values of the variables (may be changed). Unused cells are marked by „*“.

Objects may be selected by clicking their name or by right-clicking them in the geometry pane. Then they can be deleted. Curves may also be edited.

Objekt löschen | Edit | live

Name	Typ	Farbe	Fix	Sichtb...	Zeige N	Varia...	Wert1	Wert2	Wert3	Wert4
P1	point		✔	✔	✔	x,y	5,14	2,77	*	*
P2	point		✔	✔	✔	x,y	2,28	2,77	*	*
P3	point			✔	✔	x,y	5,14	2,77	*	*
C4	circle			✔	✔	x,y,r	2,28	2,77	2,86	*
L5	line			✔	✔	a,b,c	0,00	1,00	2,77	*

The equation table

The equation table may contain equations and inequalities. For the sake of simplicity, we use the word equation to denote both possibilities.
Equations may be edited and selected by clicking their number.
Right-clicking an object in the geometry pane causes all equations to be selected that contain coordinates of that object.
The **Defekt**-column gives the difference between the left hand and the right hand side of the equation. Usually, this should be 0, however, one may enter equations that have no solution. A nice example is to create to parallel lines and a point that is glued to both of the lines.
The last column contains some comments on the origin of the equation.

The Button “Simplify Eq.” tries to simplify the selected equation by applying muPads simplify and normal routines and applying some more rules.

neue Glg. | Glg. löschen | Glg. vereinfachen | Glg. anz.

Nr.	Gleichung	Defekt	Kommentar
1	a[L4]*x[P1] + b[L4]*y[P1] = c[L4]	0	P1 auf L4
2	a[L4]*x[P2] + b[L4]*y[P2] = c[L4]	0	P2 auf L4
3	a[L5]*b[L4] - a[L4]*b[L5] = 0	0	L5 parallel L4
4	a[L5]*x[P3] + b[L5]*y[P3] = c[L5]	-0	P3 auf L5
5	a[L5]*x[P6] + b[L5]*y[P6] = c[L5]	-2,817	P6 auf L5
6	a[L4]*x[P6] + b[L4]*y[P6] = c[L4]	2,817	P6 auf L4

Watching expression
Expression that do not contain =, <, > are interpreted as expressions. They have no impact on the dragging mode but their value is calculated and displayed in the defect column.

Dragging

Clicking selects the drag mode. The following kind ob objects can be dragged: points, circles, segments, vectors.
To drag a line one has to put a point on it and then one may drag the point.
To drag a curve its definition must involve some other (non fixed) objects that give it the freedom to be modified. Then one may set a point on the curve and drag the point.
There is no distinction between basic points and dependent points as in other dynamic geometry programs. However, objects may be fix or not and this influences the

Rule of Dragging: During dragging all equations and inequalities in the equation table will hold. Only variables of non-fix objects and of the object being dragged will be changed. Fix objects never move in response to other objects being moved.

In short, setting an object fix is that same as entering equations for its coordinates. i.e. if P1 is currently at 4.5, 1.8 then setting it fix has the same meaning as entering x[P1]=4.5 and y[P1]=1.8. The checkbox is just a convenient tool.

Not all drag requests may be fulfilled. FeliX tries to come as close as possible to what the user wants without violating the equations.

Other dragging modes: The modes and are stronger in the sense that one may overcome restrictions. This is useful for example if a point is bound to a higher algebraic curve that has several components. Using standard dragging one can’t leave the current component.

Constructions may get inconsistent by setting coordinates or imposing equations. The **relax tool** then tries to reach a configuration that respects all equations by moving non fixed objects.

At each time when the mouse button is released at the end of a move the current values of the coordinates (i.e. the configuration) is recorded. One may go back and forth among these saved configurations using and .

Construction tools

This section describes the tools to create new objects. The general philosophy is: First select an operation mode, then the operands.
Note that in a system like FeliX constructions like “construct midpoint between two points” can be separated into two phases: 1. Create a point. 2. Create the equations that express the geometric relation. For the user’s convenience there are however several construction buttons that combine object creation and equation generation.

Create point

Create fix point

Create midpoint
Click two points. The midpoint will be constructed

Point of intersection
Click two one-dimensional objects (lines, vectors, segments, circles, curves). Created is a point of intersection. If several intersections exist, one is selected by distance to the point of the last click.

Point on object
Click an object which will carry the point.

Segment
Click two points.

Vector
Click two points

Line though two points

Perpendicular line
Click a point and a line or vector or segment to create a perpendicular line trough the point.

Parallel line
Click a point and a line or vector or segment to create a parallel line trough the point.

Circle by midpoint and point on circumference
Click first the midpoint, then the point on the circumference

Circle with fixed radius
Click the midpoint of the circle to be constructed. The radius is taken from the input field which must contain a non negative number.

Angular bisector
Click two lines, vectors or segments.

Tools for relations

As explained above, imposing geometric relations in FeliX means imposing equations. They can be entered either directly in the equation browser or by the help of the tools described here:

Parallel
Click two linear objects (segments, vectors lines) which will be set parallel.

Orthogonal

Click two linear objects (segments, vectors lines) which will be set orthogonal.

Tangent
Click a one dimensional object (line, circle, vector, segment, curve), a point on this object and a line. The line will be set to be tangent to the first object.

Normal
Click a one dimensional object (line, circle, vector, segment, curve), a point on this object and a line. The line will be set to be normal to the first object.

Glue
Click a point and an arbitrary other object. The point will be glued to that object.

Set equal
Click two objects (segments, vectors, circles). There length (as measured by len) will be set equal.

Set length
Click an object (segment, vector, circle). Its length (as measured by len) will be set to the number in the input field.

Curves

FeliX handles different kinds of curves. Most operations on curves (intersection, setting lines tangent, etc…) work the same for all kinds of curves.

Create function graph
A dialog window appears that allows to define a function. The variable is suggested to be x but this may be changed. The expression of the function may contain variables of other objects. The function graph will then be updated when these objects change.
Definitions given in MuPAD are applied. i.e. if one has set `f:= x->x^2`
in MuPAD one may enter `f'(x)` .
The same comments hold true for the other types of curves.

Parametric curve
A parametric curve is created. One has to specify expressions for x and y depending on a parameter variable (t by default) as well as the lower and upper bound for t.

Implicit curve
Enter an equation like `x^2+y^3=5` as well as the names of the variables.

Relation curve
Click a points which is somehow restricted to one-dimensional movement. The curve is calculated and drawn.

Change curves
Curves are recorded in the table. Selecting a curve and clicking the Edit-button brings up a dialog window which allows to change the definition.

Other Operations

New
Deleted the whole construction

Zoom in / Zoom out

move viewport by dragging

Delete object
Delete an object together with all equations that involve variables of that object.

FAQs

My construction if rigid, what can I do?
This problem may several origins:
Maybe too many objects are set fixed or too many equations are entered. In this case, you have to remove constraining settings or equations.
But it may also happen that you have reached a critical configuration. An example can be illustrated using the example file panto.fx. If you drag the arm of the pantograph so that it is straight, you can go back simply by dragging. In this and similar situations you can try the following strategies:

- Use the back-button to go back to one of the last configurations.
- Use the dragging tools or .
- Go to the menu on kernel options and set the randomizing radius to some positive value like 0.01 or 0.1. This will randomize the value of the coordinated slightly before the next dragging using and hopefully you'll reach a non degenerate position.

Why is there no undo-button?
It is difficult to record all user operation because they may not only occur in the FeliX window itself but also in the MuPAD window using the text based commands of FeliX.

Why is it impossible to interrupt a long calculation of a relation curve?
The MuPAD-Java interface does not allow this. However, all calculations are automatically interrupted after one minute – so be patient.