J.-H. Eschenburg:

Comparison Theorems in Riemannian Geometry

Figures

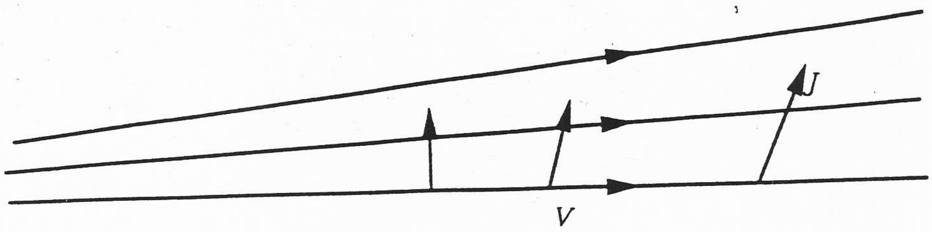


Fig. 1.

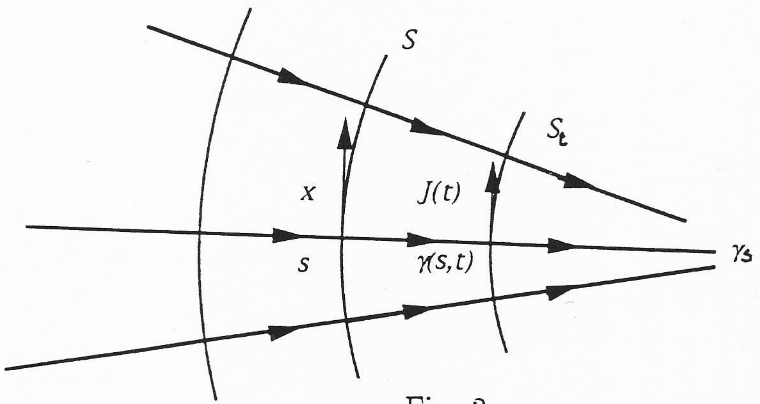


Fig. 2.

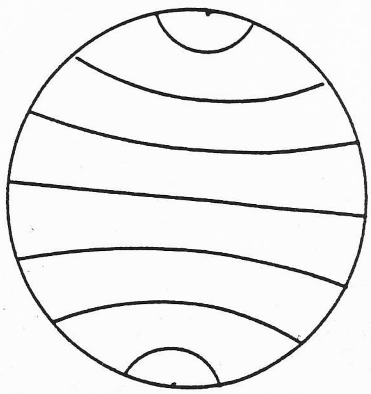
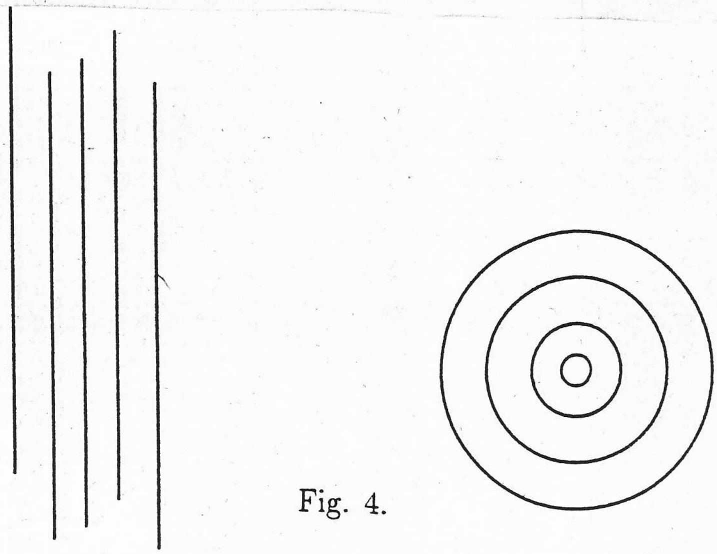


Fig. 3.



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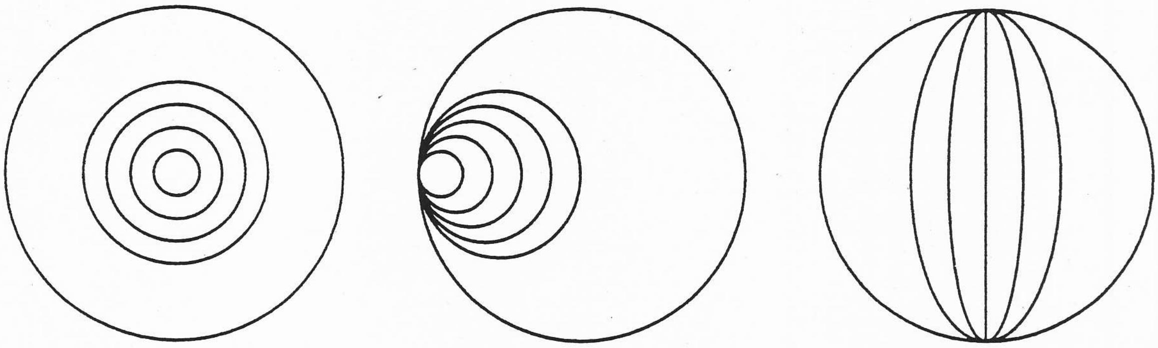

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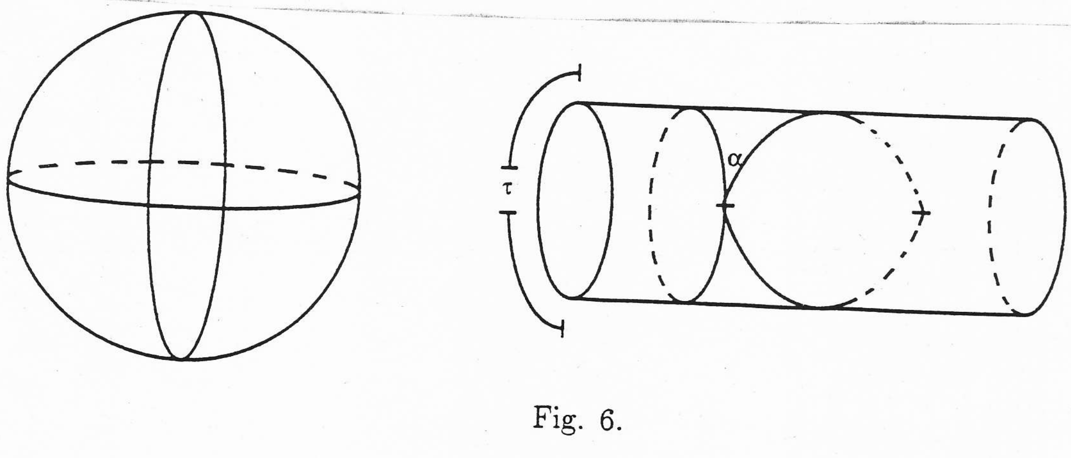

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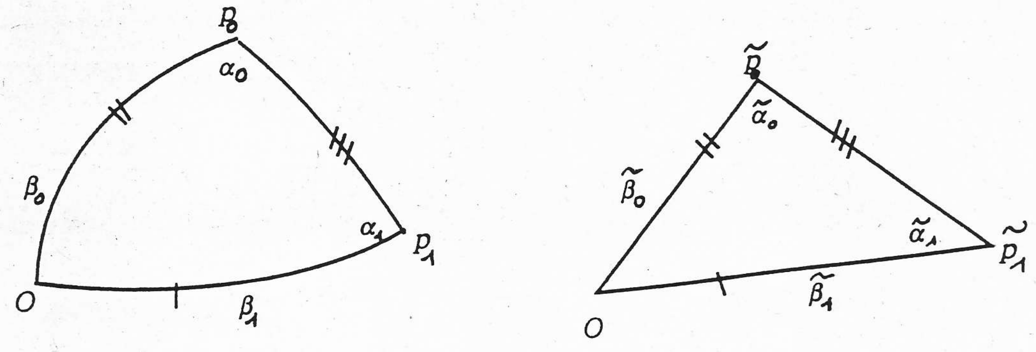

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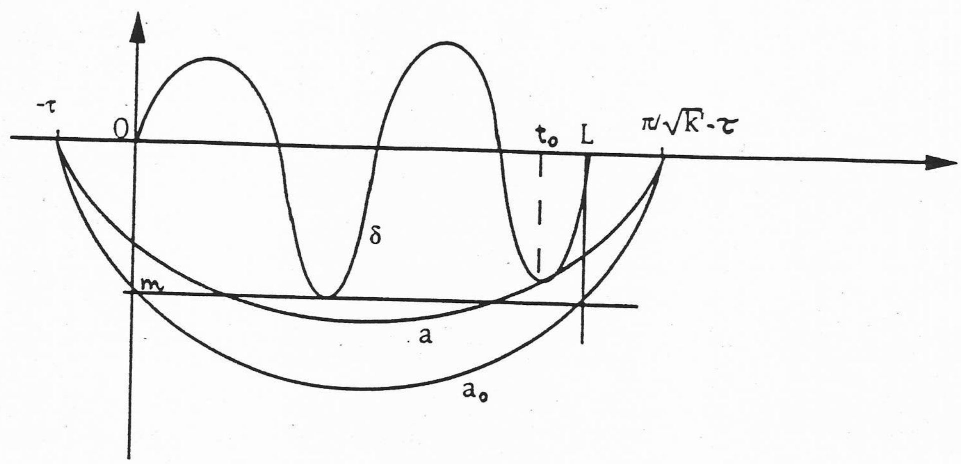

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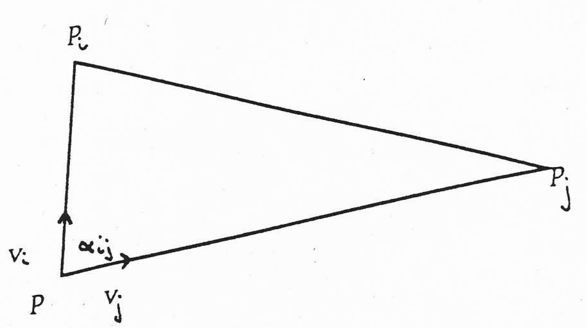

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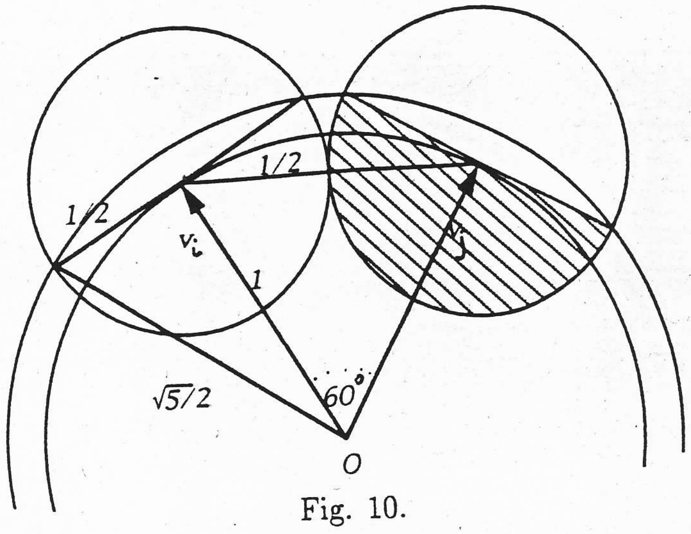

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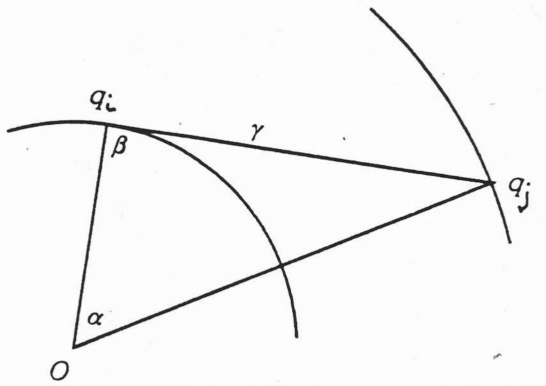

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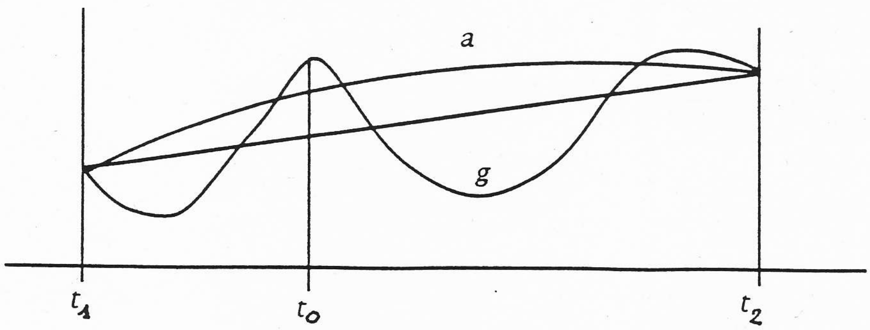

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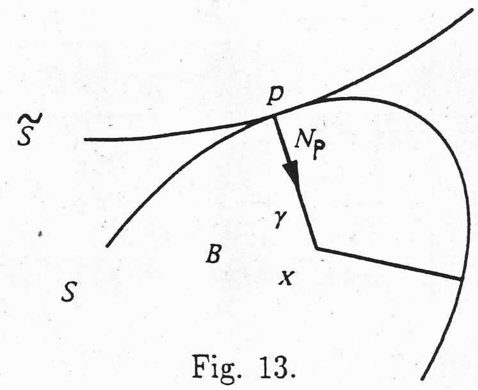

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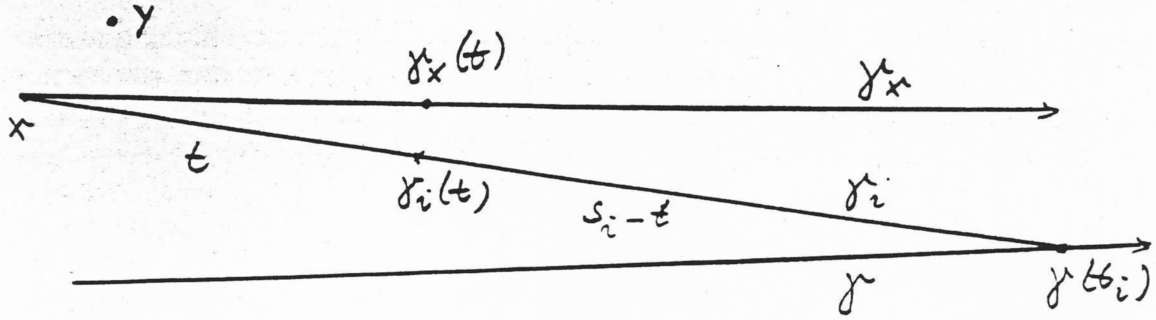

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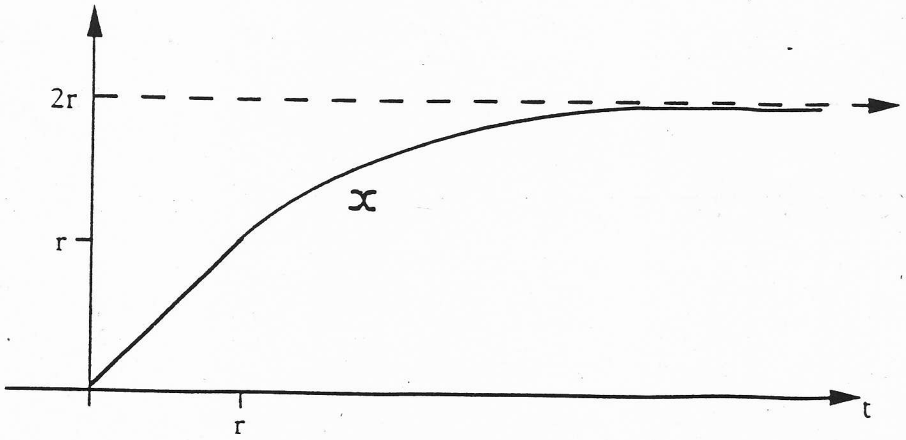

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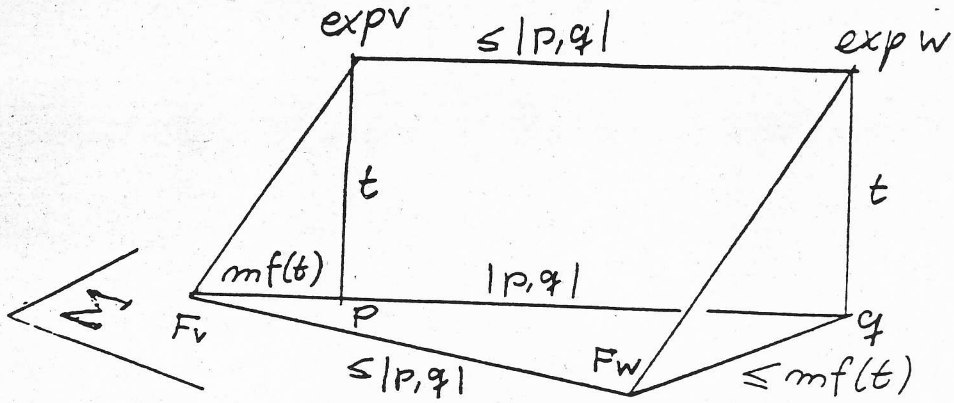

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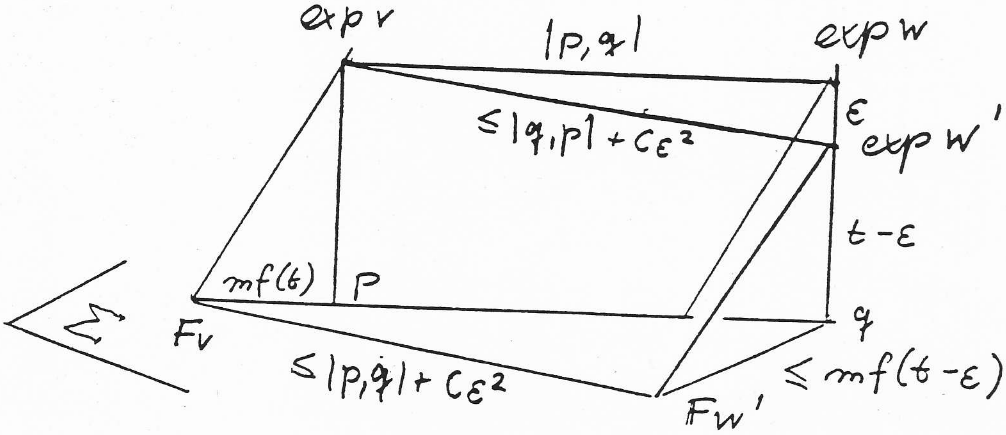

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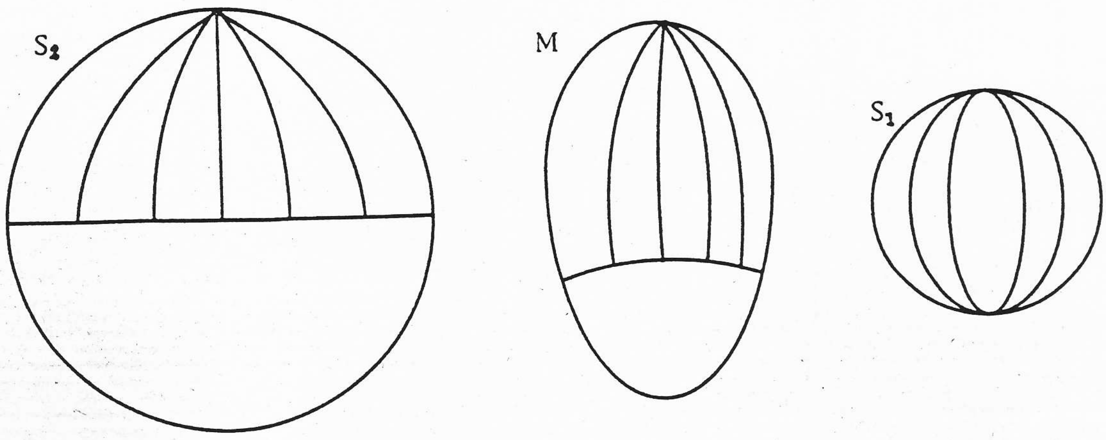

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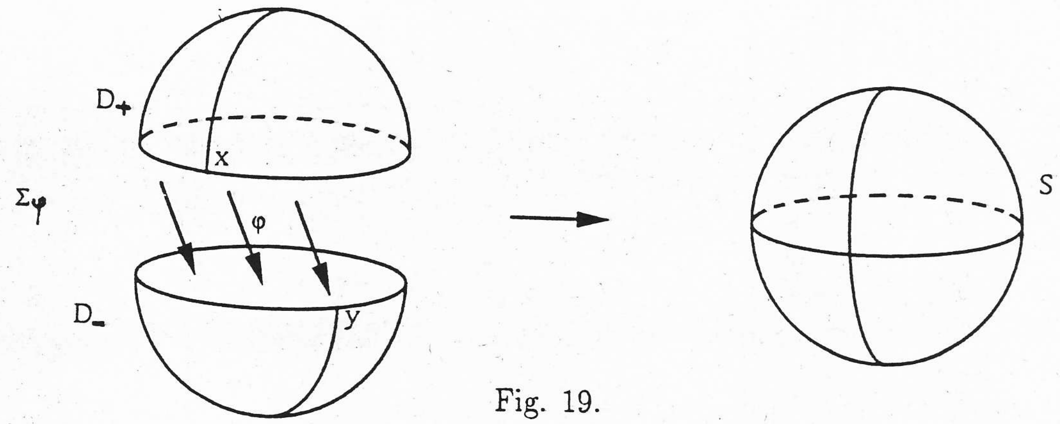

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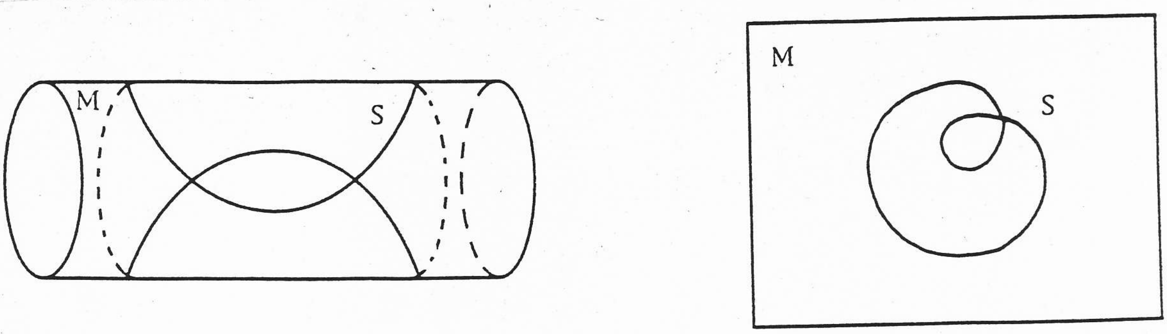

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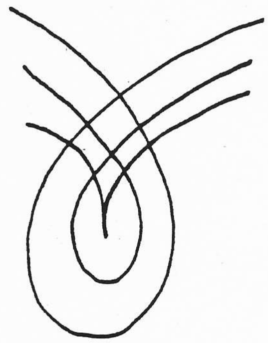

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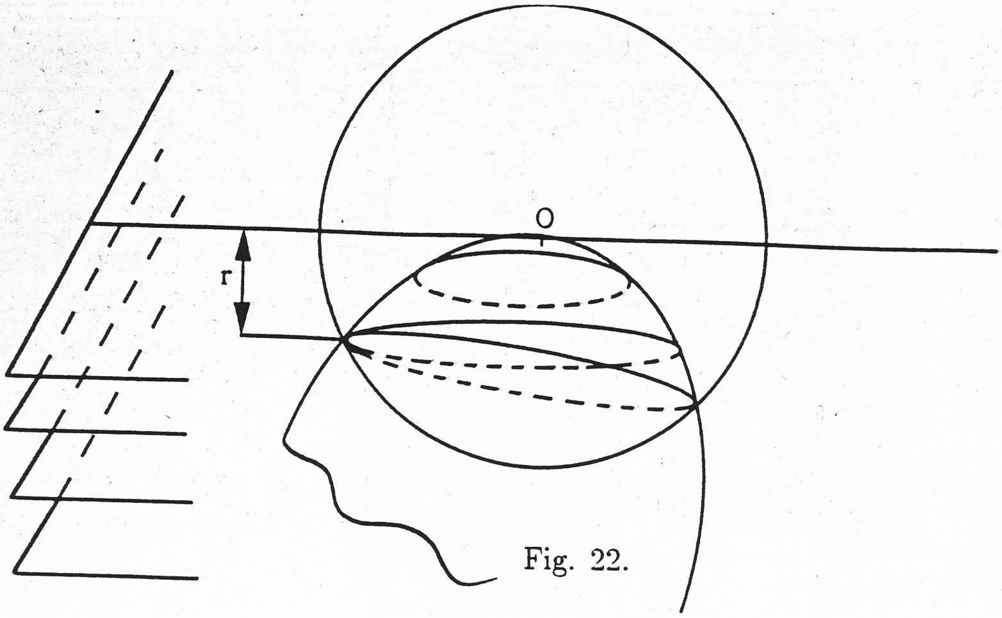

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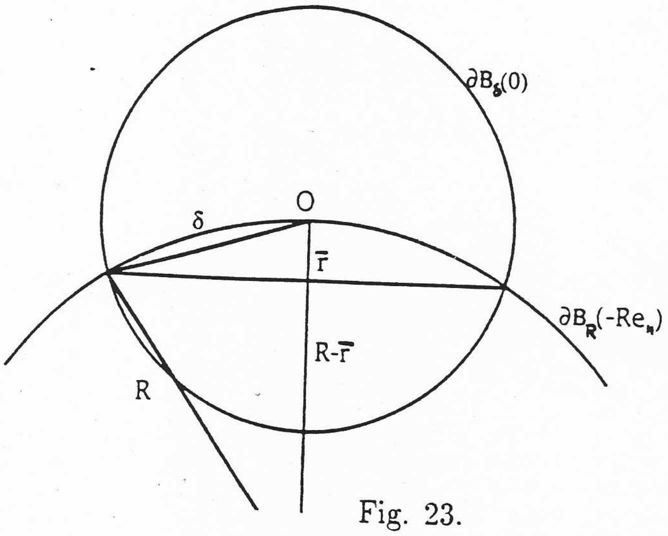

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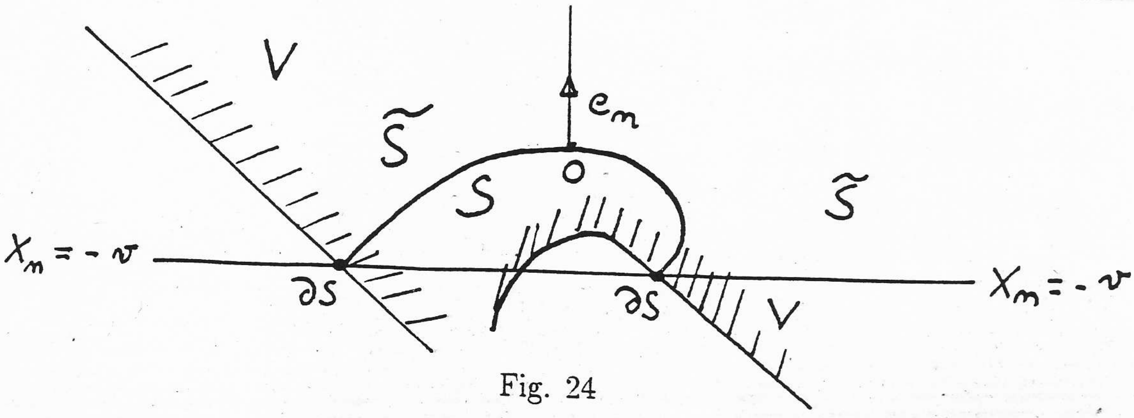

Fig. 24

second time and let $S_2 = S \setminus S_1$. All the points above S_1 are in V since they are closer to S_1 than to $\{x_n = -r\}$. Moreover, all points above S_2 are in V for the same reason.

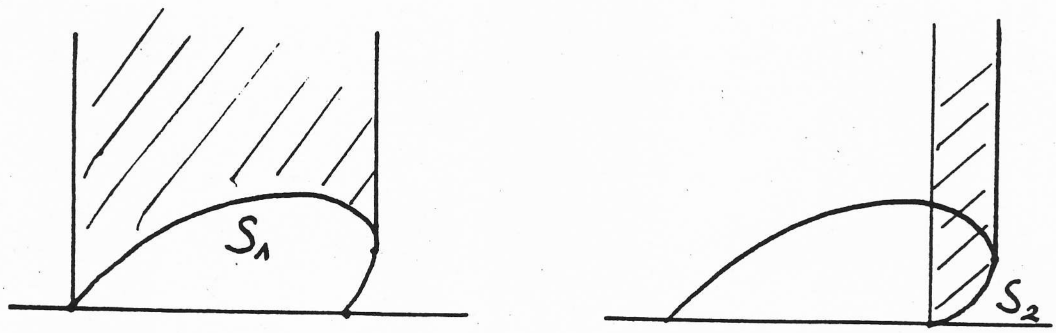

Fig. 25.

Thus we may connect any point of S to some point in $\mathbb{R}_+ \cdot e_n$ within the shaded region (cf. fig. 26 below); we just have to avoid the cylinder of height r above ∂S if we start from S_2. This finishes the proof of the claim, of the lemma and of the theorem.

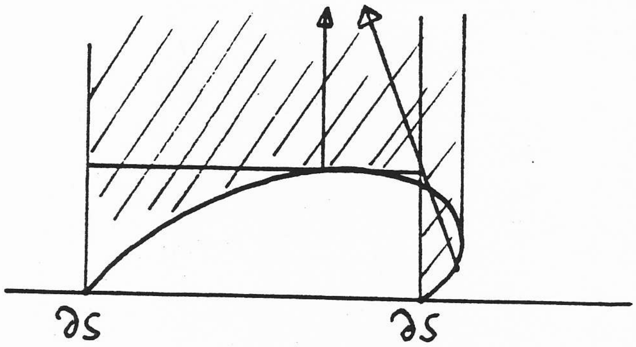

Fig. 26.

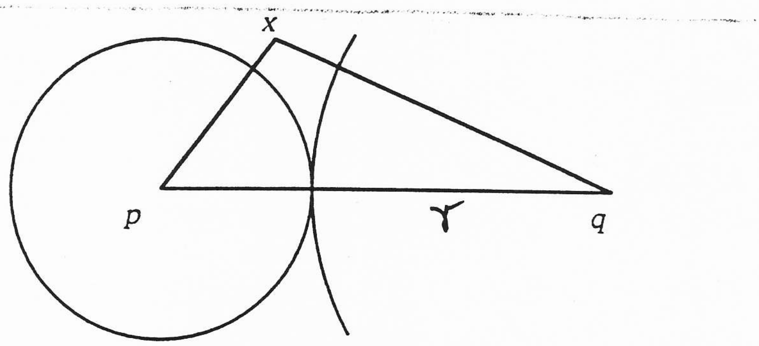

Fig. 27.

Moreover, due to $Ric \geq n-1$ and the average comparison theorem 4.1, we have on $M \setminus \{p, q\}$:

$$\Delta \rho_p \leq (n-1) \cot \rho_p, \quad \Delta \rho_q \leq (n-1) \cot \rho_q$$

in the sense of support functions. In fact, to prove the first inequality at some point $x \in M \setminus \{p, q\}$, we choose a shortest geodesic segment β from x to p and replace p by some point p' on β close to p; then the distance function $\rho_{p'}$ from p' is smooth near x and satisfies the above inequality with an arbitrary small error (by Theorem 4.1)

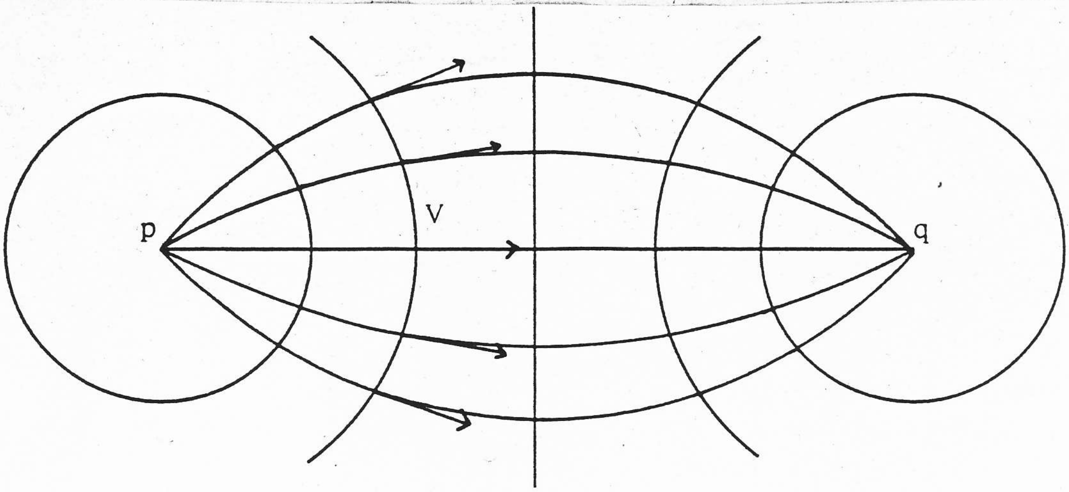

Fig. 28.