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Calculations concerning the magnetic behavior of small metallic particles at low temperatures*

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1 The problem

A calculation by Landau [1] has shown that regarding the magnetic susceptibility of metals, a diamagnetic term exists, in addition to the paramagnetism brought about by the spin of electrons. Essential for the diamagnetism is the quantization of the electron orbits due to the magnetic field. The consequences of the quantization are particularly drastic at low temperatures, when the energy kT is insufficient to bridge the level separation $\frac{e\hbar}{mc}B$ due to the quantization. According to Peierls [2] the theory yields sudden fluctuations of the magnetization as a function of the magnetic field. However, the magnitude of the magnetization is not of different order as is the case at higher temperatures.

The discovery by Meissner and Ochsenfeld [3], that a superconductor shows a diamagnetic behavior which is many times stronger than the usual diamagnetism of metals, gave rise to the question of how such strong diamagnetism could be understood by means of a model; and suggested another question of whether the exciting phenomenon of the supercurrent (without Joule heating) is perhaps in some way related to the current which has to flow in magnetic metals, according to the explanation of magnetization being due to currents. In fact there is no Joule heating associated with such an Ampère-Weber type current but, according to the usual view, in the magnetized body the current flows along closed orbits so that there is no charge flow through the entire cross section, the boundary of which lies on or outside of the surface of the magnetized body. Thus, the second question posed is whether the fact that the total current vanishes is indeed necessarily connected to the existence of a frictionless flowing current (of Ampère-Weber type).

To make the second question more precise, we make use of the concepts furnished by Maxwell's equations. The Eq. [4]

* Reprint of 'Rechnungen über das magnetische Verhalten von kleinen Metallstücken bei tiefen Temperaturen' by F. Hund, Ann. Phys. 32 (1938) 102. English translation and comment by J. Hajdu, Institut für Theoretische Physik, Universität zu Köln, D-50923 Köln, Germany.

$$\begin{aligned} \dot{B} &= -\operatorname{curl} E \\ \dot{E} &= \operatorname{curl} B - s \\ \operatorname{div} E &= \rho \\ \operatorname{div} B &= 0 \\ s &= i + \operatorname{curl} M + \dot{P} \end{aligned} \tag{1}$$

(in empty space M and P are zero) agree in content with the usual way of writing; the deviation in the form results from including the current curl M, corresponding to the magnetization in the current density (and the charge displacement leading to a change of polarization which, in the following, will not be taken into account). The contribution curl M has the property of giving rise to a vanishing total current through an entire cross section of the magnetized body, since then the line integral in

$$\int \operatorname{curl} M df = \oint M dr$$

runs within a region where $M = 0$. The fifth line of Eq. (1) is given only to show the relation to the usual formulation. To calculate the fields one has to supplement the four "basic equations" by a "material equation" for s; e.g. for many bodies

$$s = \sigma E + \operatorname{curl} \chi B + \chi E$$

holds true approximately. The second question is now if, in some way, it follows from the quantum theoretical model of a metal that the part in the material equation for s which describes a frictionless current can be cast in the form curl M.

To one of the questions posed – whether models exist which yield strong diamagnetism – an answer is insofar found, as it has been repeatedly remarked [5], that a finite quantum mechanical system of not too large size, with free moving electrons inside, can show such a strong diamagnetism due to the discreteness of its energy states. To the second question – whether the vanishing of the Joule heating necessarily implies the impossibility of a non-vanishing total current through an entire cross section of the body – an investigation by F. London [6] concerning the magnetic behavior of aromatic rings pertains; in which he shows that in thermal equilibrium a current can flow around the annulus of such a ring, though the smallness of the ring is required.

In what follows a few very simple calculations of such cases are presented. It turns out thereby that the regime of states in which small particles of metal are strongly diamagnetic has a boundary at some part of which the magnetization changes discontinuously.

2 General considerations

If we neglect the interaction among the electrons, inasmuch as it cannot be described by a static force upon each single electron, then the Schrödinger equation

$$-\frac{\hbar^2}{2m} \Delta \Psi + \frac{\hbar e}{i m c} A \operatorname{grad} \psi + \left(\frac{e^2}{2mc^2} A^2 + V - E \right) \psi = 0$$

holds for each of the electrons (charge $-e$), where

$$B = \operatorname{curl} A \ , \quad \operatorname{div} A = 0$$

is assumed for the vector potential A; V incorporates the effect of the boundaries of the metallic particle (and also that of the lattice of atomic cores if this is to be taken into account). An electron contributes to the current density by

$$s = \frac{i\hbar e}{2m}(\psi^* \operatorname{grad} \psi - \psi \operatorname{grad} \psi^*) - \frac{e^2}{mc} A \psi^* \psi \ .$$

A homogeneous field in the z-direction can be represented e.g. by a vector potential with components

$$-By, \ 0, \ 0$$

or by a vector potential with components

$$-\frac{1}{2}By, \ \frac{1}{2}Bx, \ 0$$

– the first choice being suitable if V is independent of x (an infinite wire in the x-direction); the second choice is suitable if in cylindrical coordinates (z, r, φ), V is independent of φ. In the first case one gets with

$$\psi = e^{i\varkappa x} f(y, z)$$

the equations

$$-\frac{\hbar^2}{2m}\Delta f + \left[\frac{1}{2m}\left(\hbar\varkappa - \frac{e}{c}By\right)^2 + V - E\right] f = 0$$

$$s_x = -\frac{e}{m}\left(\hbar\varkappa - \frac{e}{c}By\right) f^* f \ . \tag{2}$$

In the second case one gets, with

$$\psi = e^{i\lambda\varphi} f(z, r)$$

the equations

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2 f}{\partial z^2} + \frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r}\right) + \left[\frac{1}{2m}\left(\frac{\hbar\lambda}{r} + \frac{eBr}{2c}\right)^2 + V - E\right] f = 0$$

$$S_\varphi = -\frac{e}{m}\left(\frac{\hbar\lambda}{r} + \frac{eBr}{2c}\right) f^* f \ , \tag{3}$$

where s_φ is the component perpendicular to the plane $\varphi = \text{const}$, in the direction of increasing φ. If the resulting total current (of all electrons) has the form curl M, then one

can easily find the magnetization from the total energy ($E = \Sigma E_n$ of all electrons). Corresponding to the relation for the energy density (in certain units)

$$\eta = \frac{1}{8\pi} B^2 - \int M dB \ ,$$

where the quantum mechanical calculation yields the second term only, we obtain the magnetization $\mathfrak{M}$ of a body in a homogeneous field from

$$\mathfrak{M} = -\frac{dE}{dB} \ .$$

Since we proceed here within the electron theory, only one of the two field quantities $\boldsymbol{H}$ and $\boldsymbol{B}$ occurs in the calculations. Since we include the currents associated with the magnetization, the fundamental relation is

$$\operatorname{curl} \boldsymbol{B} = \frac{4\pi}{c} \boldsymbol{s} \ ;$$

therefore we characterize the field by $\boldsymbol{B}$.

Single small metallic particle

We consider a piece of metal of finite size and rotational symmetry in a homogeneous magnetic field (B = const) in the direction of the symmetry axis, and idealize the piece of metal by a potential $v(z,r)$. According to (3) the energy of a single electron is

$$E(\lambda, n_z, n_r; B) = \frac{e\hbar}{2mc} \lambda B + F(|\lambda|, n_z, n_r; B^2)$$

where F does not depend on the sign of λ. At absolute zero temperature, in the minimum of the total energy, these states are occupied up to a sharp limit. Let the number of electrons be such that for $B = 0$ in the uppermost occupied state (which should not accidentally correspond to $\lambda = 0$) both components $\lambda = |\lambda|$ and $\lambda = -|\lambda|$ are occupied. Then, for very small values of B, the term in the total energy which is due to λB drops out and the magnetization

$$\mathfrak{M} = -\frac{dE}{dB}$$

is diamagnetic. Due to the factor $8c^2$ in the denominator, the susceptibility will be of the order 10^{-5} if the sample of metal has the size of an atom and a few electrons only. The susceptibility will increase in the same degree as the size (and $\overline{r^2}$) grows, if the number of electrons per unit volume is kept fixed; it will be of the order 1 when the cross section reaches the value of 10^5 atomic cross sections. This holds true for very small fields only. Above a certain field B we obtain two electrons more with negative λ as with positive λ; suddenly a constant positive magnetization adds to the negative

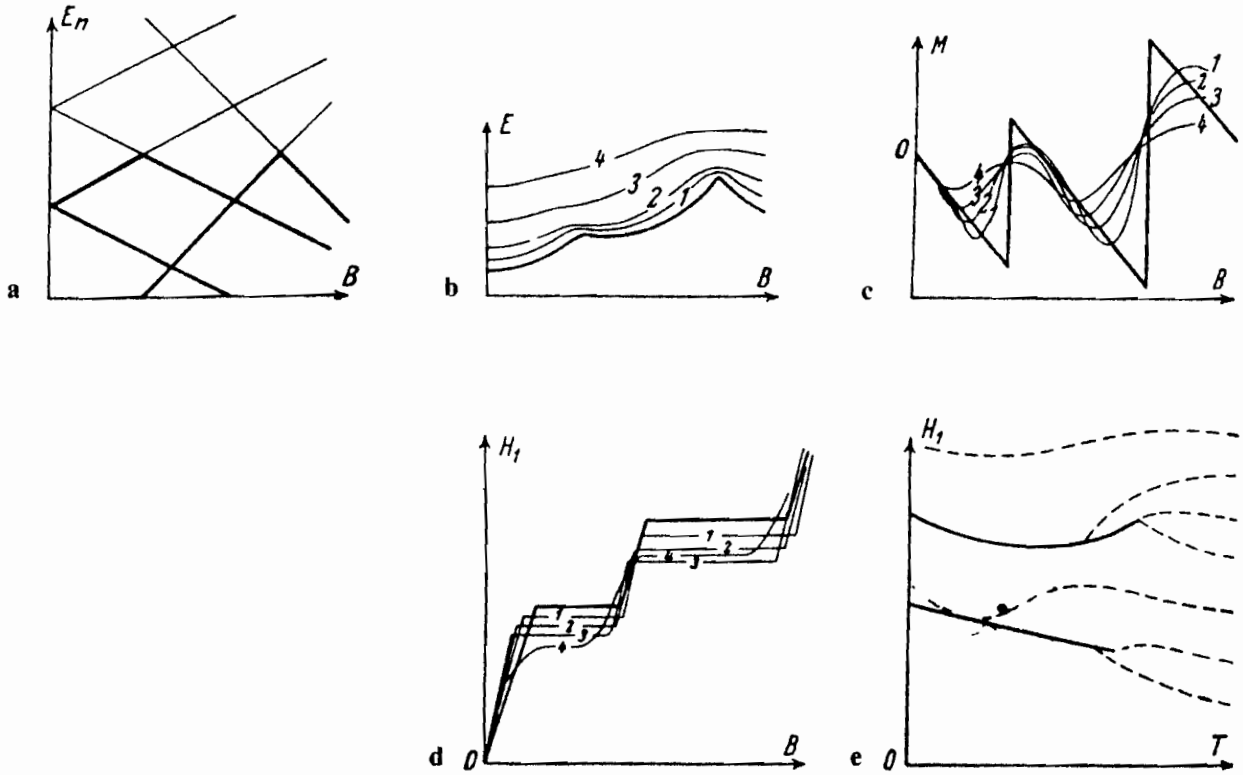

Fig. 1a–e **a** Example for the scheme of energy levels of an individual electron. **b** Total energy. **c** Magnetic moment. **d** Relation between magnetic induction and external field. **e** Phase diagram

(diamagnetic) magnetization, and this repeats after a certain increment of B. Fig. 1a shows a possible term distribution, the states represented by thick lines are occupied at $T=0$, the thick line in Fig. 1b shows the total energy as a function of B and the thick line in Fig. 1c shows the magnetization.

The details of these results depend on the particular random properties of the levels (spacing and λ) at the edge of occupation. However, the order of magnitude of the discontinuity in $\mathfrak{M}$ is the same as the value of $|\mathfrak{M}|$.

If λ_1 and λ_2 are the $|\lambda|$-values of the levels which cross at the first jump of $\mathfrak{M}$, then the total energy is, up to an irrelevant constant,

$$E=\frac{e^2}{8mc^2}\Sigma\overline{r^2}B^2+\begin{cases}0\\ D-\dfrac{e\hbar}{2mc}B(\lambda_1+\lambda_2)\end{cases}$$

and the magnetization is

$$\mathfrak{M}=-\frac{e^2}{4mc^2}\Sigma\overline{r^2}B+\begin{cases}0\\ \dfrac{e\hbar}{2mc}(\lambda_1+\lambda_2)\end{cases}$$

If R is the radius, and L the length of the body of rotational symmetry considered above, respectively, then the number of electrons is of order R^2L/a^3 (a being the atomic unit of length, $\hbar^2/me^2$), and the orders of magnitude of the terms in our equation are given by

$$\Sigma\overline{r^2} = 4\alpha\frac{R^4L}{a^3} \; ; \quad \lambda_1 + \lambda_2 = 2\beta\frac{R}{a}$$

$$D = \gamma\frac{a^3}{R^2L}\frac{me^4}{\hbar^2}$$

where α, β, γ are of the order 1. This yields

$$\frac{\mathfrak{M}}{e\hbar/mc} = -\alpha\frac{R^4L}{a^5}\frac{B}{cm^2e^3/\hbar^3} + \begin{cases} 0 \\ \beta\dfrac{R}{a} \end{cases}$$

The magnetization jumps at

$$\frac{B}{cm^2e^3/\hbar^3} = \frac{\gamma}{\beta}\frac{a^4}{R^3L}$$

from

$$\frac{\mathfrak{M}}{e\hbar/mc} = -\frac{\alpha-\gamma}{\beta}\frac{R}{a}$$

to

$$\frac{\mathfrak{M}}{e\hbar/mc} = \left(-\frac{\alpha\gamma}{\beta}+\beta\right)\frac{R}{a}$$

The atomic unit of field

$cm^2e^3/\hbar^3$ is $2.35\cdot 10^9$ Gauss.

For example, to obtain a magnetization corresponding to $\frac{M}{B} = 1$, a level spacing D of about 10^{-5} atomic energy units or 3 k degrees (k is the Boltzmann constant) and a critical magnetic field of about 10^{-7} atomic field units or 200 Gauss, one has to choose R to be about 100 and L about 10 atomic radii.

We now have to keep in mind that a measurement of the magnetic behaviour does not measure the dependence of the moment $\mathfrak{M}$ on the "internal" field B but the dependence of the quantity $\mathfrak{M}$, M or B on an external field which we will denote by H_1. In the case of a long body, H is inside and outside the body practically equal to H_1, and consequently

$$H_1 = B - 4\pi M(B) \; ; \tag{5}$$

in the case of another shape, a numerical factor has to be attached to H_1. If, for example, M/B is very strongly negative, then this means a very small ratio B/H_1. The limit of strong diamagnetism is $B = 0$ [7]. Already $\frac{M}{B} = -1$ means $B = \frac{1}{1+4\pi}H_1$, i.e. "almost $B = 0$."

To obtain $B(H_1)$ we have to perform an appropriate affine transformation of the $M(B)$ diagram related to Eq. (5). Strictly speaking, in our case M is not constant (as

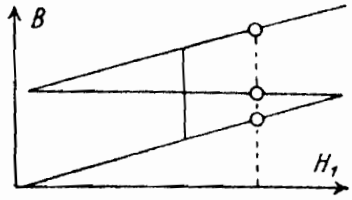

Fig. 2 Multivalued $B(H_1)$

follows for example from the calculation of the current); the error we make when replacing M by our moment $\mathfrak{M}$ divided by the volume is insignificant as long as the susceptibility is small in comparison to unity; however our diagrams are only qualitatively valid if the susceptibility is comparable with unity. Since, however, treating an inhomogeneous $\boldsymbol{B}$ is inconvenient, we will leave it at that. The $B(H_1)$ curve for $T=0$ then takes the form shown in Fig. 2. For certain values of H_1 there exist three solutions for B and a thermodynamic consideration, as occurs in connection with the van der Waals equation of state and in investigations of ferromagnetism, shows that the probable values belong to a $B(H_1)$ curve, a part of which is parallel to the B axis and cuts off two plain pieces of equal area from the original curve.

If we abandon, for a moment, the condition $T=0$, and take T and H_1 as variables of state, then the behavior will be determined by

$$dS = \frac{dE - H_1\,dM}{T} \;;$$

at fixed T and H_1 the most probable value is the one with the smallest

$$Z = E - TS - MH_1 \;,$$

and this leads to the vertical connection mentioned above. For smaller values of H_1 an (irreversible) transition can occur from the highest value to the lowest of the three values of B, for higher H_1 an (irreversible) transition from the lowest to the highest; along the vertical connection the transition is reversible.

Under certain circumstances in the scheme of levels it can also happen that the highest B value is more probable already at $H_1 = 0$.

If a body (supposed to be much larger than an atom) shows, at $T=0$, strong diamagnetism, then *the regime of this diamagnetism is bounded by a threshold value of the external magnetic field at which B jumps from a very small value to a larger one.*

In order to investigate the *effect of temperature* we assume our quantum mechanical system to be in contact with a heat bath of a given temperature and consider the time average of the magnetization, more precisely: we assume for the single electron occupation the probability

$$\frac{1}{1 + e^{\frac{E-\zeta}{kT}}}$$

corresponding to Fermi statistics; and express in terms of this, the total energy as a function of B and T, the magnetization and finally $B(T,H)$. This is performed graphically in Figs. 1b, 1c, 1d; the thin curves 1, 2, 3, 4 correspond to temperatures

larger than zero. Depending on the random properties of the levels in Fig. 1a, the threshold of the field H_1 increases or decreases with T. It may happen that the limiting $H_1(T)$ curve terminates at a value $H_1 \neq 0$ and then a continuous but quick transition occurs from small to larger B values. It may also happen that the discontinuous curve extends to $H_1 = 0$. The order of magnitude of the temperature at which the transition of B to larger values occurs, is of course given by $\Delta E/k$ where ΔE is the level spacing for $B = 0$ at the occupation edge. Fig. 1e shows the curve of discontinuity and another one at higher H_1 values; the dashed curves are lines of $B = \text{const}$.

So far we required rotational symmetry for the metallic particle under consideration, since in this case the calculations are very simple. The essential qualitative features remain, however, also in the general case of a small sample of metal. At $T = 0$ and small B the diamagnetic effect dominates. To the same degree that the eigenfunctions of the electrons change with increasing B, more and more occupied states occur with negative average angular momentum around the field axis, giving rise to a paramagnetic effect. The $M(B)$ curve shows thereby no spikes, since the average of the total angular momentum around the field axis changes continuously.

The influence of the electron spin can be incorporated easily into the calculations and does not change anything essential.

4 Mixture of small metallic particles

When assuming a mixture of small metallic particles such as the one just considered, then, of course, the sizes and the sets of energy levels are somewhat different. If no

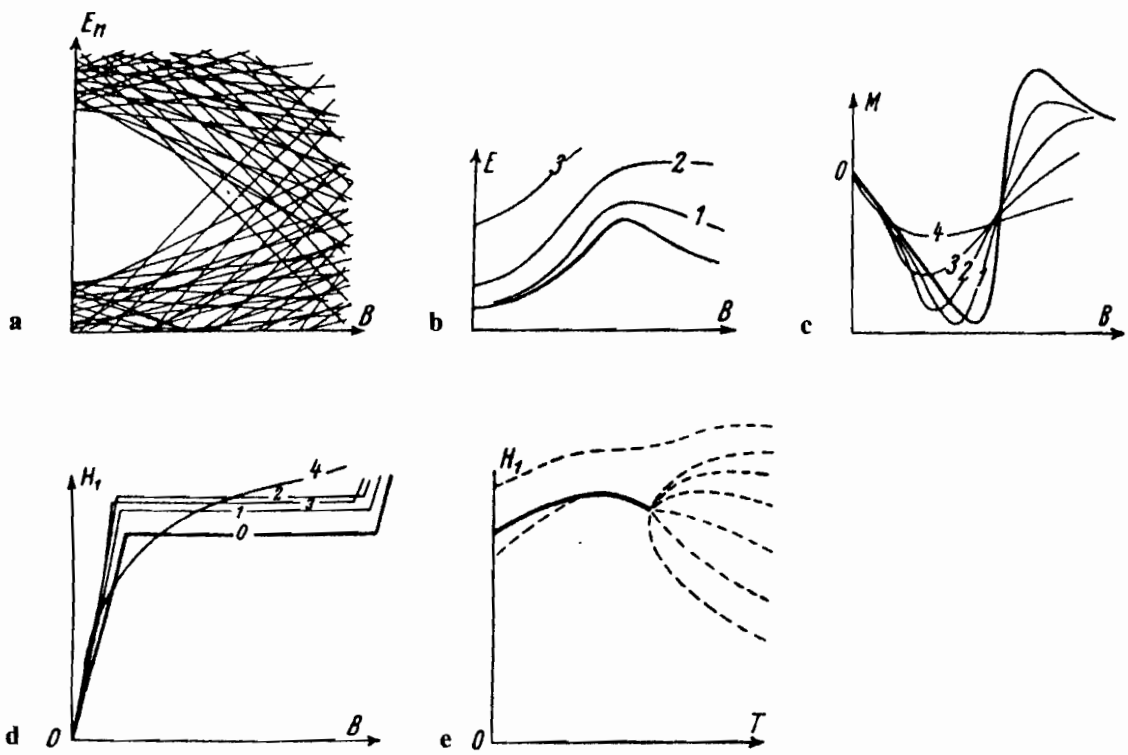

Fig. 3a – e **a** Example for the structure of single electron energy levels of a mixture. **b** Total energy. **c** Magnetic moment. **d** Relation between magnetic induction and external field. **e** Phase diagram

special selection has taken place, all ΔE below an upper limit, determined by the minimal size, occur uniformly (since E depends on three quantum numbers in different ways). Thus, for $T = 0$, the positions of B where the magnetization jumps are uniformly distributed; the size of the jump depends on the values of λ and hence is not systematically correlated with the position of the jump. Then diamagnetic and paramagnetic terms average themselves out.

The situation is different if we prepare mixtures by specific selection, e.g. using the magnetic behavior of the particles, which are assumed to be mobile, to prepare a mixture of particles which are strongly diamagnetic in a certain range of low temperatures [8]. Placing the systems of levels (corresponding to Fig. 1 a) of these small metallic particles on top of each other, a region at small remains free, whereas around that region a rather uniform level density prevails. Constructing $B(H_1)$ we get, at low temperatures, something similar to that of a single piece; the jumps at higher temperatures, however, disappear, and we obtain there approximately $B = H_1$. An example of this type is carried out in Fig. 3a to 3e. The result can be changed somewhat by varying the boundary of the hole in the scheme of levels shown in Fig. 3.

5 Currents without energy dissipation

As pointed out by London [6], the diamagnetism of aromatic ring molecules is connected with a current around the hole of the ring; this current is not accompanied by energy dissipation since it belongs to the state of lowest energy.

Our calculations for a small sample of metal lead to the same conclusion if the sample is assumed to be shaped as a *ring*. Expression (4) for the energy of a single electron remains, in this case, true. For each electron

$$S_\varphi = -\frac{e}{m}\left(\frac{\hbar\lambda}{r}+\frac{eBr}{2c}\right)f^*f$$

yields the contribution

$$\int S_\varphi dr dz = -\frac{e}{2\pi m}\left(\hbar\lambda\frac{\bar{1}}{r^2}+\frac{eB}{2c}\right)$$

to the current through a cross section extending outward from the hole of the ring. At $T = 0$ and sufficiently weak B field, $\lambda = |\lambda|$ and $\lambda = -|\lambda|$ are equally strongly represented, so that the current

$$-\frac{e^2 B}{4\pi m c}n$$

follows, where n is the number of electrons; it flows around the hole. It represents, for $T = 0$, a state of equilibrium. In fact, a usual conduction current would have at $T = 0$, in an ideal lattice, no associated resistivity either; however, since it would not be a state of equilibrium, a small perturbation would be sufficient to dissipate the energy. As soon as the magnetic field in our ring at $T = 0$ is increased such that a jump in the

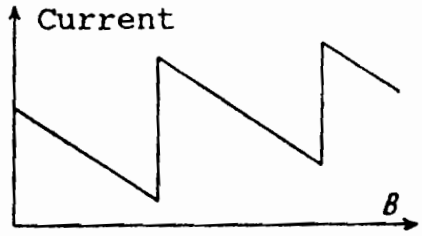

Fig. 4. Current around the annulus of a ring

magnetization occurs, an additional current opposite to the previous one appears around the hole due to the preference of negative values of λ, most of which flows at the inner edge; the total current around the hole is in general not zero but depends rather on the accidental measures of the ring. This total current around the hole as a function of B looks similar to that given in Fig. 4.

For $T>0$, in thermal equilibrium, negative λ values are preferred also at small B; this, however, has no particular consequence for small T so that an essential part of the current corresponding to the diamagnetic magnetization flows around the ring; this current causes no Joule heating, since the occupation of states corresponds to thermal equilibrium. Of course, another current can also flow around the hole, but in this case the states are occupied in a different way and each perturbation (e.g. the interaction with the metal lattice) leads to dissipation of energy. Thus, in the case of small rings at low temperatures, the current density giving rise to the magnetization cannot be cast in the form curl M. If we wish to keep the formal relation $s = i + \operatorname{curl} M$, we have to attach this current to i, in spite of its "Ampère-Weber type character".

In the case of larger rings the magnetization practically ceases already at weak fields, due to the small level spacing. The current around the hole decreases even more rapidly than that appropriate for the magnetization, since this is due to a current which flows in the outer part of the ring in one direction and in the inner part in the opposite direction around the hole.

As the single example considered above already shows, the division of the current into two parts,

$$i + \operatorname{curl} M$$

where i is associated with energy dissipation, is not strictly valid, but represents an approximation sufficient for practical purposes.

Let us consider for example a straight wire with constant cross section in a homogeneous magnetic field perpendicular to the wire. This case corresponds to Eq. (2) where $V(x,y)$ is due to the boundary of the wire. Sufficiently distant from the boundary, the eigenfunction f is determined, apart from E, only by the term $\frac{1}{2m}\left(\hbar\varkappa - \frac{e}{c}By\right)^2$; in particular f^*f is symmetric relative to the plane $\hbar\varkappa - \frac{e}{c}By = 0$. The expression for s_x shows that already in this single electron state, the same amount of current flows forwards in one direction as it does backwards in the other. Here the current of even a single electron can be described by curl M. Near the surface of the wire the situation is different. A potential V which increases in the y direction shifts f^*f to locations where $\hbar\varkappa - \frac{e}{c}By > 0$; thus such states have a total current in the

$-x$ direction [9]. On the opposite surface of the wire there exists a current in the $+x$ direction. If $V(y)$ is symmetric then these currents cancel each other out exactly in the total current (not, however, in the magnetic moment, as shown by Landau's calculation).

The dismantling of the equation $s = i + \operatorname{curl} M$, with only the second term representing the frictionless current, by the counter-example of a small ring at low temperatures is (just because of the smallness of the ring) still negligible. However, for larger bodies, we could not give a general proof for the equation; in fact it might not strictly hold. In spite of the similarity of the examples considered with the behavior of superconductors, it is not yet obvious if the former can contribute anything to the explanation of the latter. To explain this behavior in the framework of a model we still have to overcome two essential difficulties. We have to understand how the diamagnetic behavior of an extended body is brought about. And even if we could explain this, perhaps by a breaking off in small diamagnetic domains caused in some way by the interaction between the electrons, the second difficulty of understanding a current which carries charge through an entire cross section of an extended conductor [10] remains.

6 Summary

Considerations concerning small metallic samples of suitable size yield, at low temperatures and weak magnetic fields, a region of states with strong diamagnetism. This region is partially bounded by locations of discontinuity of the magnetization. In this respect such metallic samples are similar to superconductors.

In the case of small metallic rings a current around the hole arises simultaneously with a strong diamagnetism, as shown in an investigation by London. The usual separation of the electric current into a dissipative conduction current and a current of Ampère-Weber type with no dissipation, and also no total current, throughout an entire cross section, is for all practical purposes an approximation of very high accuracy.

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[8] "Take" a porous powder of very small metallic particles and sort them with the help of an inhomogeneous magnetic field at very low temperatures (of course, concerning the technical possibilities of the experimental realization I do not want to make any comments)

[9] Similar to the state of electrons in an electric field, S. Titeica, Ann. Phys. (Lpz.) [5] **22**, 129 (1935)

[10] Within London's formulation of the phenomenological approach, the two difficulties take the following form: first one must clarify whether in the case of singly connected bodies, at approximate choice of $\boldsymbol{A}$ the relation $\Lambda \boldsymbol{i} = -\boldsymbol{A}$ holds ("diamagnetism"), and second, if in the case of multiple connected regions the way London utilizes the ambiguity of the equation $\Lambda \operatorname{curl} = \boldsymbol{i} = -\operatorname{curl} \boldsymbol{A} = -\boldsymbol{B}$ is indeed the one which corresponds to reality ("superconductivity").

Translator's note

The work on the magnetic properties of small metallic samples is one of the last research papers published by Friedrich Hund. It is a pioneering investigation in the field which is now called the physics of mesoscopic systems. In the first part of the paper Hund discusses the large diamagnetism as being due to the single electron energy level structure which is characteristic for small metallic particles in a magnetic field. In the second part he makes a few important remarks concerning the interrelation between magnetization, Ampère-Weber type currents and vanishing dissipation. Thus, Hund's work is relevant for both types of mesoscopic systems: small particles and small size conductors.

From the technical point of view Hund's paper is a masterpiece of qualitative analysis, based on simple model calculations and graphic sketches.

It is rather difficult for us today (60 years later) to trace back the reception of the paper when it was first published. Reference to it can neither be found in the standard monographs nor in the related works by Peierls and Kubo – although both of them had knowledge of the paper. Presumably the first reference to the paper after World War II was made by R.B. Dingle, Proc. Roy. Soc. (London), Ser. A212, 47 (1952). [cf. also R.V. Denton, Z. Physik **265**, 119 (1973)]. Recently von Oppen drew attention to Hund's work [PhD Thesis, Univ. of Washington, 1993].

Friedrich Hund has his own personal style of scientific writing. We tried to preserve it as much as this is possible in English. The system considered in the paper is referred to by him as small pieces of metal (kleine Metallstückchen). For this we used the more up-to-date terms of small metallic particles and small metallic samples. In the second part of the paper the term "mesoscopic conductors" would have been more appropriate but an unacceptable anachronism.

In the formulas Gothic types for vectors have been replaced by bold faced characters. $\mathfrak{M}$ denotes the volume integral of $\boldsymbol{M}$.

The text is unabridged and unchanged. Only a few minor mistakes have been corrected.

The translator is indebted to Dr. Susi Krebs and Dr. Tim Newman for valuable support and to Dr. Felix von Oppen for providing him with a copy of his PhD thesis.

Editorial: Friedrich Hund – 100 Years

The 'last page' in each issue of Annalen der Physik contains a reprint of the journal's table of contents of 100 years ago. The present issue shall in addition commemorate another event: The 100th birthday (4 February) of Friedrich Hund, Professor Emeritus in Göttingen. His name is familiar to every student of physics and chemistry by things like 'Hund's rules' and the 'Hund-Mulliken method'.

In 1938 Hund published a paper with the title 'Rechnungen über das magnetische Verhalten von kleinen Metallstücken bei tiefen Temperaturen' in this journal (Ann. Physik 32 (1938) 102). There he studies size effects of magnetic properties in metals, and the paper is a very interesting precursor of today's mesoscopic physics. The first article in this issue is a reprint of Hund's work in an English translation kindly done by Professor J. Hajdu.

B. Mühlschlegel