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EXPERT OPINION

Full counting statistics in mesoscopic fermion and boson systems

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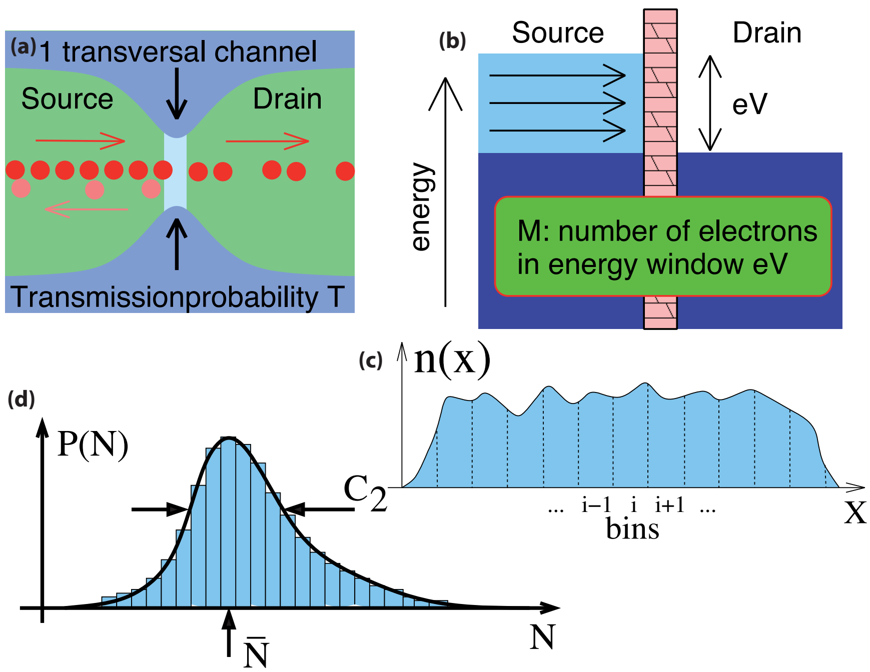

Figure 1 Two examples demonstrating the use of the counting statistics concept. In a quantum point contact (a) electrons impinge on a barrier in a constriction, some are reflected and some transmitted in a random manner. The number of incoming electrons M in a time interval t can be understood in a energy diagram (b). The number is related to the energy window spanned by the applied bias voltage eV and given by $M = \mathrm{eV}t/h$. As the particles are independent the statistics of the total transmitted number is binomial. In a similar way we can discuss the spatial fluctuations of the particle density, e.g. in cloud of ultra cold atoms, depicted symbolically in (c). By considering a cell or a bin defining a certain spatial region, the particle number in it is a stochastic quantity and follows from some probability distribution. Such a distribution (d) typically centered around some mean value represents the complete information on the system available by particle number measurements and, hence, this is called the full counting statistics. (Figs. a and b modified from Physik Journal 4, 75–80 (2005); Figs. c and d modified from [7]).

Quantum mechanics is an inherently stochastic description of physical systems. In particular, the outcome of a measurement is probabilistic and many repetitions of the same experiment reveal fluctuations of the observable. It is important to recall that this is the case although the state of the systems is fully determined and described by a deterministic time evolution. Of course, an uncertainty in the state preparation, e.g., due to a large number of degrees of freedom also leads to fluctuations, but these are less fundamental from a quantum perspective (of course they are of importance in all practical systems, which are, e.g., at a finite temperature). The quantum fluctuations, which remain after all sources of uncertainty have been eliminated, are of central interest in the mesoscopic physics of ultracold electrons and atoms.

One of the most important quantities in quantum transport processes is the current through a quantum point contact, a small constriction connecting two large fermionic reservoirs with an electric potential difference V. Assuming the transmission probability to be T, one finds for the average transferred charge $\langle Q\rangle$ in a time period t at zero temperature $\langle Q\rangle = e^2 V T t/h$. Here, e is the electron charge and h denotes Planck's constant. It is also straightforward to calculate the fluctuations of the transferred charge [1,2]

$$\langle \Delta Q^2 \rangle = e^3 |V| T(1-T) t/h$$
$$= (1-T)|\langle Q \rangle|.$$

These formulas have a simple interpretation as average and variance of an underlying binomial statistics. For uncorrelated particles one would obtain $\langle \Delta Q^2 \rangle_{cl} = |\langle Q \rangle|$, which follows in the limit of small transmission $T \ll 1$. The vanishing fluctuations for $T \rightarrow 1$ are a consequence of the Fermi statistics of electrons. It should be emphasized that the fluctuations just discussed are pure quantum fluctuations because they result from the particles having evolved into a superposition state of being on each side of the contact simultaneously.

Obviously, average and variance alone do not determine the full probability distribution of the transferred charge. For mesoscopic systems, this full counting statistics was first addressed by Levitov and Lesovik [3]. They also made the interesting observation that, in order to calculate the statistics of the accumulated current, one has to consistently account for the quantum mechanical measurement process. For simple quantum point contacts they confirmed that the statistics is binomial. Later, Nazarov and coworkers put this on a more solid ground and showed that the charge counting statistics can, in general, be related to the time-evolution of the state of the current detector [4]. However, an interesting conceptual question in that context arises since the interpretation of the counting statistics in terms of a probability is not guaranteed to work. In fact, it was found that in the case of a contact between superconductors, the counting statistics apparently leads to the puzzling result of seemingly negative probabilities [5].

In their paper, Rammer and Shelankov [6] give a pedagogical introduction to the interpretation problem and describe their solution to this problem. By introducing a particle tagging with the help of a spatially selective gauge transformation which does not disturb the quantum dynamics, they are able to extract the statistics of the charge number in the selected space region. Whether or not the resulting generating function can be interpreted as a probability of charge transfer events depends on initial coherences, viz. whether a particle is in a superposition of being in the selected region and outside. This alternative derivation of the Levitov-Lesovik formula for the charge transfer statistics is very elegant. It also clarifies the limitations of its applicability and explains why its naive application to superconducting junctions cannot be straightforwardly interpreted as a charge transfer probability. Furthermore, the formalism of Rammer and Shelankov can be applied to track a quantum measurement in real time, e.g., by a spin coupled to the charge transfer. They observe the emergence of two time scales. On the shorter time scale, the off-diagonal charge state decays and, consequently, the problem of interpreting the counting statistics becomes insubstantial. After a longer time scale, the charge distributions have been separated and one can say the spin state has been measured. Finally, Rammer and Shelankov apply their formalism to number fluctuations in interfering Bose-Einstein condensates [7]. It is straightforward to generalize their counting gauge transformation to a local cell inside a cloud of ultracold atoms. It naturally reproduces the experimental observation that in each snapshot of two independent Bose-Einstein condensates an interference pattern is observed, the phase of which is, however, random.

Summarizing, one can say that the field of full counting statistics has already brought up many surprises. The connection to quantum measurement theory opens up a new perspective, also in the light of coherently controlling quantum systems. Many more surprises are likely to come up on a fundamental level, and quantum measurements and their counting statistics may become relevant for high-level applications like, e.g., quantum feedback.

Key words. Quantum noise, full counting statistics.

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