Theoretical Concepts and Simulations for Materials Scientists Summer term 2009

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Exercises 2

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1. Consider the Gauß function
$$G_\eta(x-x_0) = \frac{1}{\eta\sqrt{\pi}} e^{-\left(\frac{x-x_0}{\eta}\right)^2} \qquad \eta > 0\,.$$
Sketch the Gauß function, determine its maximum, and calculate the width of the Gauß curve, which is defined by the distance between the two points $x_0 \pm \alpha$, where
$$G_\eta(x \pm \alpha - x_0) = \frac{1}{\eta\sqrt{\pi}} \frac{1}{e}\,.$$
Finally, calculate the integral
$$I = \int_{-\infty}^{+\infty} dx\, G_\eta(x-x_0)\,.$$
Consider especially all results for the limit $\eta \to 0$.

2. Consider the Lorentz function
$$L_\eta(x-x_0) = \frac{1}{\pi} \frac{\eta}{(x-x_0)^2 + \eta^2} \qquad \eta > 0\,.$$
Sketch the Lorentz function, determine its maximum, and calculate the width of the Lorentz curve, which is defined by the distance between the two points $x_0 \pm \alpha$, where the function assumes half of the maximum value. Finally, calculate the integral
$$I = \int_{-\infty}^{+\infty} dx\, L_\eta(x-x_0)\,.$$
Consider especially all results for the limit $\eta \to 0$.

3. In one dimension, Dirac's δ-distribution is specified by the requirements

   (a)
$$\delta(x-x_0) = 0 \qquad \forall x \neq x_0$$

   (b)
$$\int_a^b dx\, \delta(x-x_0) = \begin{cases} 1 & : \quad a < x_0 < b \\ 0 & : \quad \text{elsewhere} \end{cases}$$

Show that

(a)

$$\lim_{\eta \to 0} G_\eta(x - x_0) = \delta(x - x_0)$$

(b)

$$\lim_{\eta \to 0} L_\eta(x - x_0) = \delta(x - x_0)$$