

All-Electron Full-Potential Calculations at $\mathcal{O}(\text{ASA})$ Speed

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Outline

- 1 Background
- 2 Full-Potential ASW Method
- 3 Proof of Concept: Results



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Back in the 1930's ...

John C. Slater



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Full Potential

$$v_{\sigma}(\mathbf{r}) : \begin{cases} \text{spherical symmetric near nuclei} \\ \text{flat outside the atomic cores} \end{cases}$$



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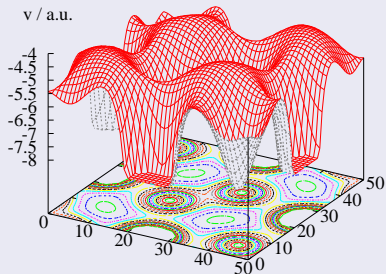
Muffin-Tin Approximation

$$v_{\sigma}^{MT}(\mathbf{r}) = \begin{cases} \text{spherical symmetric in spheres} \\ \text{constant in interstitial region} \end{cases}$$



Back in the 1930's ...

Full Potential (FeS₂)



Muffin-Tin Potential



Back in the 1930's ...

Muffin-Tin Approximation

distinguish:

- atomic regions

- remainder

Muffin-Tin Potential



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distinguish:

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 - $v_{eff,\sigma}(\mathbf{r}) = v_{eff,\sigma}(|\mathbf{r}|)$
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 - $v_{eff,\sigma}(\mathbf{r}) = 0$

Muffin-Tin Potential



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Partial Waves

- muffin-tin spheres
 - $v_{eff,\sigma}(\mathbf{r}) = v_{eff,\sigma}(|\mathbf{r}|)$
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 - solve radial Schrödinger equation numerically
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 - exact solutions
 - plane waves
 - spherical waves

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- match at sphere surface („augment“)

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Basis Functions

- matched partial waves
 - augmented plane waves (APWs)
 - „muffin-tin orbitals“ (MTOs), augmented spherical waves (ASWs)



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Wave Function

expand in basis functions

- expansion coefficients from variational principle

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Core States

all-electron methods

- fully included
- orthogonal to partial waves

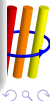
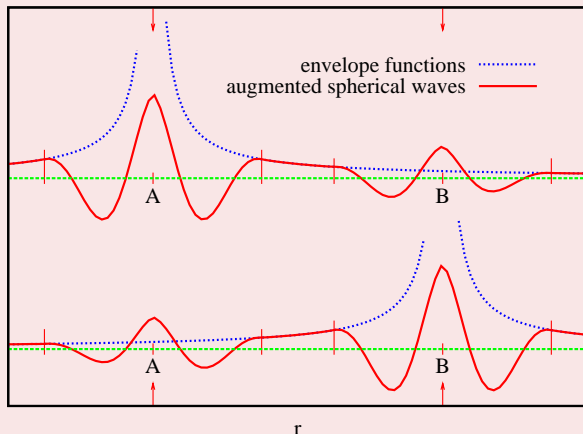
Basis Functions

- matched partial waves
 - augmented plane waves (APWs)
 - „muffin-tin orbitals“ (MTOs), augmented spherical waves (ASWs)
- used to describe valence states



Back in the 1930's ...

Augmented Spherical Waves



Back in the 1970's ...

Ole K. Andersen



“Linear Methods in Band Theory”

- energy dependence of basis functions almost linear \rightarrow linearize $(\varphi, \dot{\varphi})$
 - huge increase in computat. efficiency!



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Linear Augmented Plane Wave (LAPW)

- muffin-tin approximation
- easy to implement good!
- full-potential at a low price
 - basis functions from muffin-tin potential
 - wave functions from full potential
 - example: Wien2k
- large basis set (≈ 100 pw's/atom) bad!

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Linear Muffin-Tin Orbital (LMTO)

- based on spherical waves
 - does not require crystalline periodicity
 - natural interpretation of results
- difficult to implement **bad!**
- **full-potential extension extremely difficult**
- muffin-tin approximation (?)
 - finite interstitial region
 - large basis set:
two functions per s -, p -, d -state
 - **still inefficient** **bad!**

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 - **make spheres space-filling!**
 - **interstitial region formally removed**
 - **only numerical functions in spheres**
 - **minimal basis set (s , p , d)**
 - **very high computational efficiency**
→ $\mathcal{O}(\text{ASA})$ speed!!!
 - makes potential more realistic
 - **systematic error in total energy** bad!

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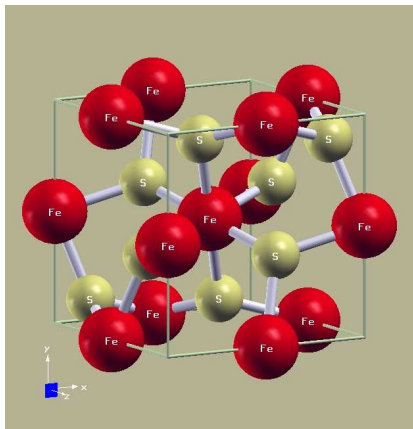
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Iron Pyrite: FeS_2



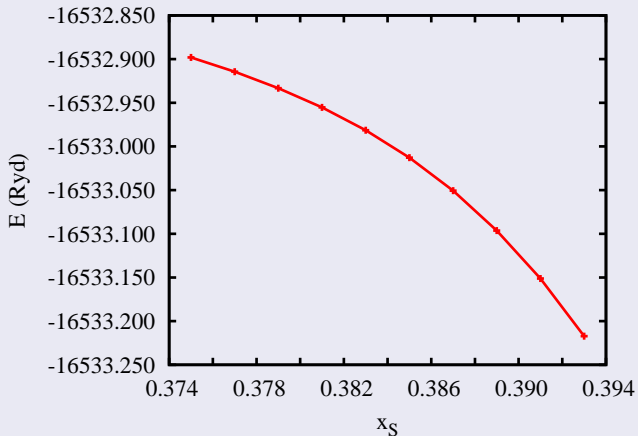
Pyrite

- $Pa\bar{3} (T_h^6)$
- $a = 5.4160 \text{ \AA}$
- “NaCl structure” sublattices occupied by
 - iron atoms
 - sulfur pairs
- sulfur pairs $\parallel \langle 111 \rangle$ axes
- $x_S = 0.38484$
- rotated FeS_6 octahedra



FeS₂: Structure Optimization

ASA⁺ Code



Towards a Full-Potential Spherical-Wave Method

Conclusions

- ASA (space-filling atomic spheres)
 - $\mathcal{O}(\text{ASA})$ speed
 - systematic error in total energy
- non-overlapping muffin-tin spheres
 - prerequisite for accurate total energies
 - larger basis set \rightarrow inefficient



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Requirements

- restore interstitial region
 - go to non-overlapping muffin-tin spheres
 - go beyond constant-potential approximation
- inside muffin-tin spheres
 - non-spherical contributions



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Guidelines

- interstitial quantities expanded in **plane waves**
 - straightforward to implement
 - inefficient
- interstitial quantities expanded in **spherical waves**
 - elegant, no periodicity required
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 - difficult to implement



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Characteristics

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 - different linearization scheme
 - different interstitial energy
 - different implementations



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0th Generation (Williams, Kübler, Gelatt, 1970s)

PRB **19**, 6094 (1979)



ASW Method

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- “dialect” of LMTO
 - different linearization scheme
 - different interstitial energy
 - different implementations

1st Generation (VE, 1990s)

IJQC 77, 1007 (2000)

- completely new, monolithic implementation
- new algorithms → improved accuracy, numerical stability
- much improved functionality, usability, and portability
- xAnderson convergence acceleration scheme
- all LDA-parametrizations, most GGA-schemes
- still based on atomic-sphere approximation



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Basic Principles

Steps to be Taken

- **remove total energy error** due to overlap of atomic spheres
 - reintroduce non-overlapping muffin-tin spheres
 - restore interstitial region



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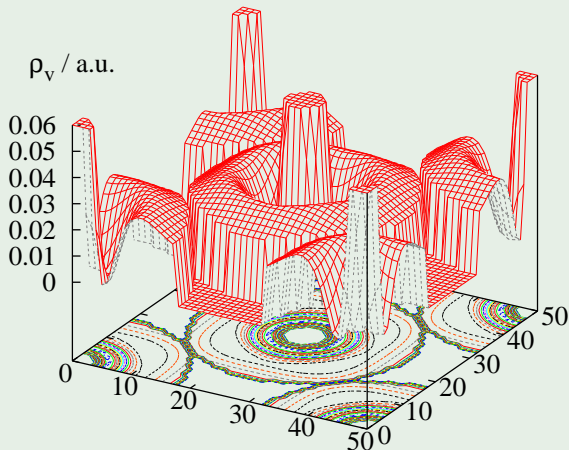
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- find representation of **products of the basis functions**
 - inside muffin-tin spheres
 - **use spherical-harmonics expansions**
 - in the interstitial region
 - **no exact spherical-wave representation available!**



From Wave Functions to Electron Density

Density inside MT-Spheres

(A1)



From Wave Functions to Electron Density

Products of Basis Functions in Interstitial Region

$$\rho^l(\mathbf{r}) = \sum_n d_n F_n(\mathbf{r})$$

$$\int d^3\mathbf{r} F_{n'}^*(\mathbf{r}) \rho^l(\mathbf{r}) = \sum_n d_n \int d^3\mathbf{r} F_{n'}^*(\mathbf{r}) F_n(\mathbf{r})$$



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- $F_n(\mathbf{r})$: plane waves
 - integrals exact
 - inefficient
 - Weyrich 1988, Blöchl 1989, VE 1991, Savrasov 1992, Methfessel 2000



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- $F_n(\mathbf{r})$: **spherical waves**
 - would be efficient
 - integrals not known analytically
 - Springborg/Andersen 1987, Methfessel 1988, VE 2002, VE 2006



From Wave Functions to Electron Density

Interstitial Products of Basis Functions

expand in atom-centered spherical waves (Hankel functions):

$$p^l(\mathbf{r}) := (H_{L\kappa_1\sigma}^\infty(\mathbf{r}_i))^* H_{L'\kappa_2\sigma}^\infty(\mathbf{r}_j) \stackrel{!}{=} \sum_n \sum_{K\eta} d_{K\eta n\sigma}^{L\kappa_1 i L'\kappa_2 j} H_{K\eta}(\mathbf{r}_n)$$

- $H_{L'\kappa_2\sigma}^\infty(\mathbf{r}_j)$: orbital basis set (obs)
- $H_{K\eta}(\mathbf{r}_n)$: product basis set (pbs)

coefficients from projection \mathcal{P} :

$$\begin{aligned} \mathcal{P} [p^l(\mathbf{r})] &= \mathcal{P} [(H_{L\kappa_1\sigma}^\infty(\mathbf{r}_i))^* H_{L'\kappa_2\sigma}^\infty(\mathbf{r}_j)] \\ &= \sum_n \sum_{K\eta} d_{K\eta n\sigma}^{L\kappa_1 i L'\kappa_2 j} \mathcal{P} [H_{K\eta}(\mathbf{r}_n)] \end{aligned}$$



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- Springborg/Andersen 1987
 integrate over interstitial region, use one-center expansions

$$\mathcal{P}_{K'\eta'n'} \dots \hat{=} \int_{\Omega_I} d^3\mathbf{r} H_{K'\eta'}^*(\mathbf{r}_{n'}) \dots$$



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- Methfessel 1988
 match values and slopes at MT-sphere surfaces

$$\mathcal{P}_{K'\eta'n'} \dots \hat{=} \int d^2\hat{\mathbf{r}} Y_{K'}(\hat{\mathbf{r}}) \left(\frac{\partial}{\partial r} \right)^{\eta'-1} \dots \Big|_{|\mathbf{r}_{n'}|=s_{n'}}$$



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Interstitial Products of Basis Functions

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- VE 2002
 integrate over interstitial region, use one-center expansions
 (interstitial \rightarrow shells between AS- and MT-spheres)

$$\mathcal{P}_{K'\eta'n'} \dots \hat{=} \sum_m \int_{\Omega'_m} d^3\mathbf{r}_m H_{K'\eta'}^*(\mathbf{r}_{n'}) \dots$$



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experience

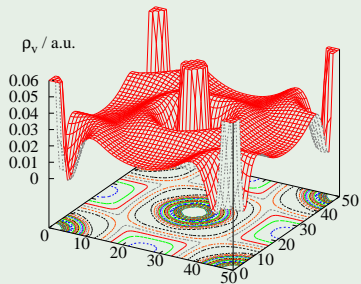
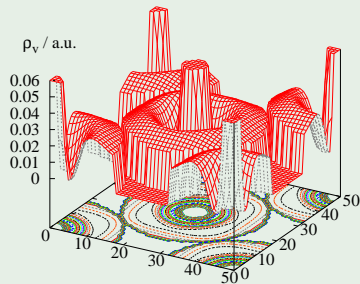
- product basis set almost overcomplete
- value/slope (Methfessel) matching most stable



From Wave Functions to Electron Density

Density from Value/Slope Matching at MT-Radii

(A)



From Electron Density to Full Potential

Inside Muffin-Tin Spheres

- density, Hartree-potential and xc-potential numerically

Interstitial Region

- density from value/slope matching
- Hartree-potential analytically
- xc-potential from value/slope matching



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From Full Potential to Basis Functions

Previous Approaches

- project full potential to **muffin-tin potential**
- construct basis functions from **muffin-tin potential**
- **no minimal basis set!** (large basis set!)

Present Approach

- project full potential to **ASA potential**
- construct basis functions from **ASA potential**
- **minimal basis set!**
- future
 - use spheres larger than space-filling
 - remove linearization error



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Comparison of Approaches

Ole K. Andersen

- ASA geometry used for basis functions
→ minimal basis set good!
- ASA geometry used for density and potential
→ error in total energy bad!



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Michael S. Methfessel

- MT geometry used for density and potential
→ accurate total energy good!
- MT geometry used for basis functions
→ large basis set bad!



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present approach

- ASA geometry used for basis functions
→ minimal basis set → $\mathcal{O}(\text{ASA})$ speed great!
- MT geometry used for density and potential
→ accurate total energy great!



Implementation

2nd Generation ASW (VE, 2000s)

- based on 1st generation code
- full-potential ASW method
 - electron densities, spin densities
 - electric field gradients
 - elastic properties, phonon spectra
- optical properties
 - based on linear-response theory
 - direct calculation of $\Re\sigma$ and $\Im\sigma$
 - no Kramers-Kronig relations needed
- LDA+U method
 - all „flavours“ for double-counting terms (AMF, FLL, DFT)

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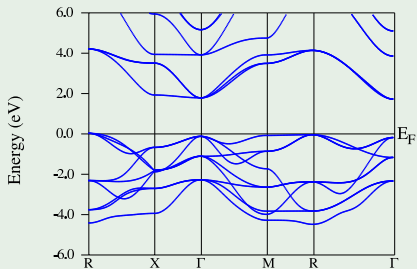
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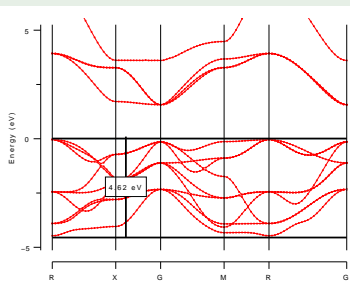


Electronic Structure of BaTiO₃

Wien2k

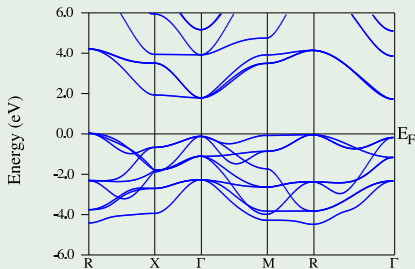


VASP

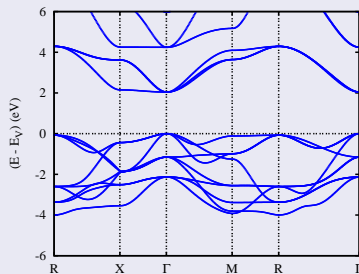


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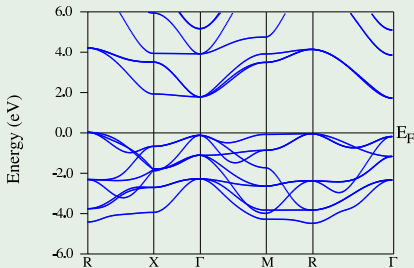


ASA+ Code

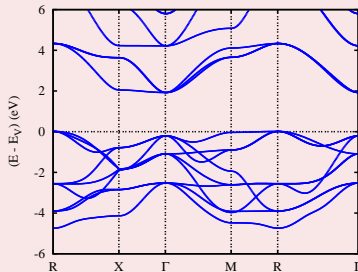


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Wien2k



Full-Potential Code



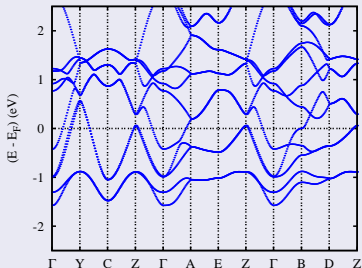
New!

- much better agreement with other full-potential codes (valence-band width, valence states at M-point)

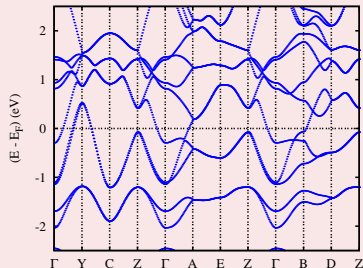


Fermi Surface of MoO_2

ASA Code



Full-Potential Code

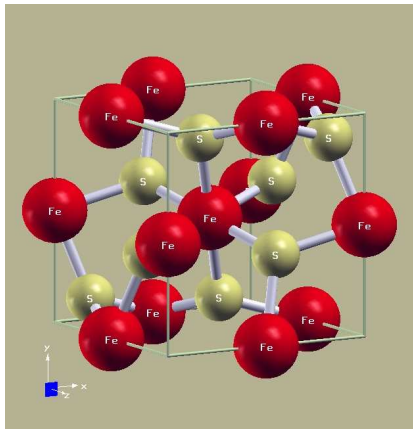


New!

- no hole pocket near Z-point
- much better agreement with ARPES, de Haas-van Alphen



Iron Pyrite: FeS_2



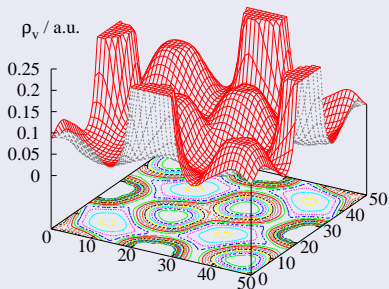
Pyrite

- $Pa\bar{3} (T_h^6)$
- $a = 5.4160 \text{ \AA}$
- “NaCl structure” sublattices occupied by
 - iron atoms
 - sulfur pairs
- sulfur pairs $\parallel \langle 111 \rangle$ axes
- $x_S = 0.38484$
- rotated FeS_6 octahedra



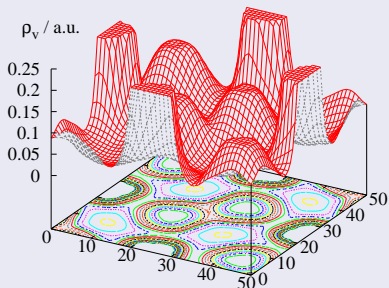
FeS₂: Density and Laplacian

Valence Electron Density

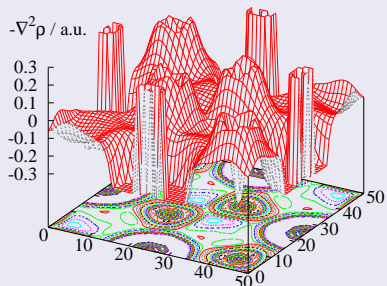


FeS₂: Density and Laplacian

Valence Electron Density



Laplacian of Electron Density

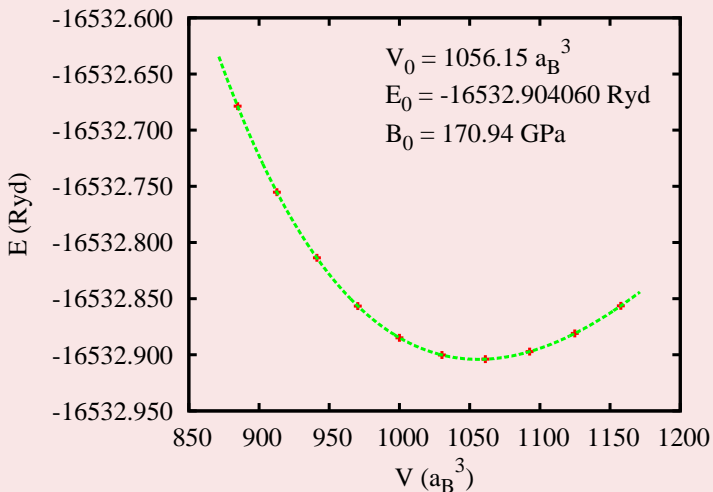


New!

- topological analysis (Bader analysis)



FeS₂: Equilibrium Volume and Bulk Modulus



FeS₂: Equilibrium Volume and Bulk Modulus

Lattice Constant

10.28	NCPP	Zeng and Holzwarth '94
10.02	FPLO	Opahle <i>et al.</i> '99
10.17	CRYSTAL98	Muscat <i>et al.</i> '02
9.92	CASTEP	Muscat <i>et al.</i> '02
10.18	FPASW	present work
10.23	exp.	Finklea <i>et al.</i> '76
10.22	exp.	Will <i>et al.</i> '84
10.23	exp.	Stevens <i>et al.</i> '91



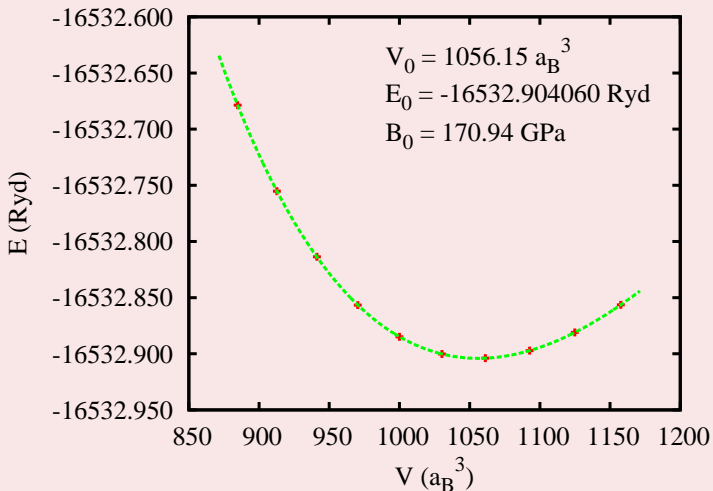
FeS₂: Equilibrium Volume and Bulk Modulus

Bulk Modulus

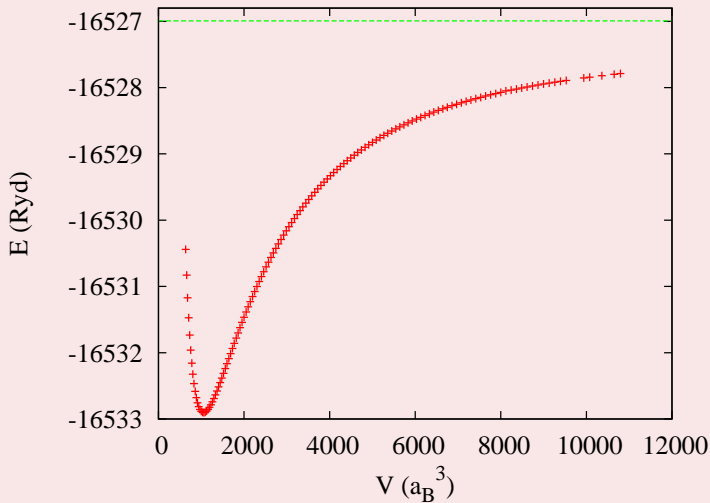
187	LMTO	Nguyen-Manh <i>et al.</i> '98
185	FPLO	Opahle <i>et al.</i> '99
209	CRYSTAL98	Muscat <i>et al.</i> '02
208	CASTEP	Muscat <i>et al.</i> '02
171	FPASW	present work
148	exp.	Drickamer <i>et al.</i> '66
118	exp.	Will <i>et al.</i> '84
215	exp.	Chattopadhyay and von Schnering '85
157	exp.	Fujii <i>et al.</i> '86
143	exp.	Jephcoat and Olson '87
162	exp.	Ahrens and Jeanloz '87
145	exp.	Blachnik <i>et al.</i> '98



FeS₂: From Atoms to the Solid

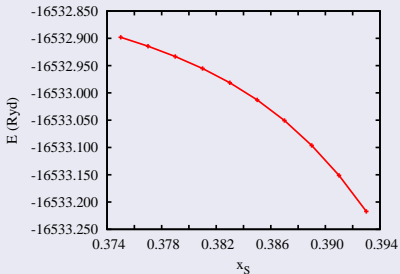


FeS₂: From Atoms to the Solid



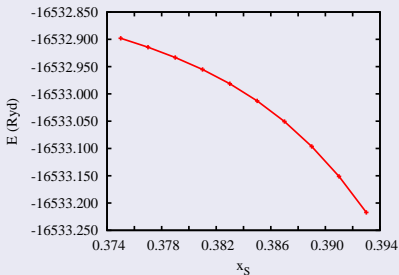
FeS₂: Structure Optimization

ASA+ Code

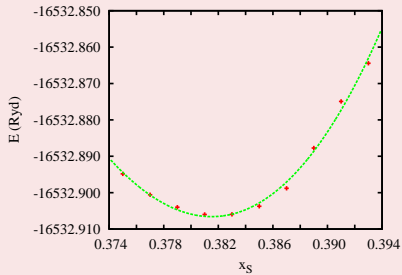


FeS₂: Structure Optimization

ASA⁺ Code



Full-Potential Code



FeS₂: Structure Optimization

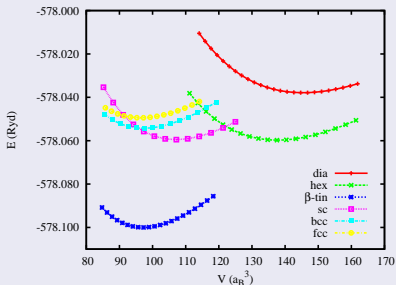
Sulfur Position

0.378	NCPP	Zeng and Holzwarth '94
0.377	FPLO	Opahle <i>et al.</i> '99
0.378	CRYSTAL98	Muscat <i>et al.</i> '02
0.382	CASTEP	Muscat <i>et al.</i> '02
0.382	FPASW	present work
0.386	exp.	Finklea <i>et al.</i> '76
0.386	exp.	Will <i>et al.</i> '84
0.385	exp.	Stevens <i>et al.</i> '91



Phase Stability in Silicon

ASA+ Code



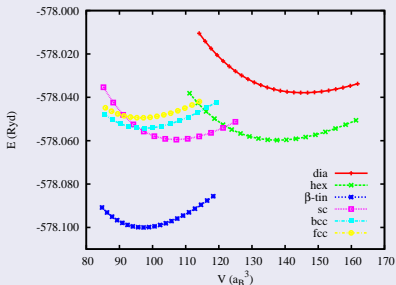
Bad

- β -tin structure most stable # nature (diamond structure)

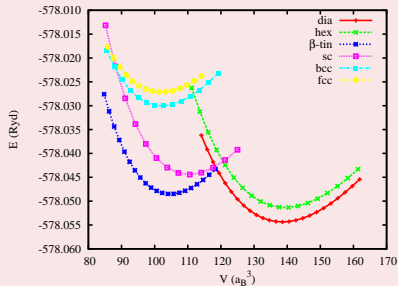


Phase Stability in Silicon

ASA⁺ Code



Full-Potential Code



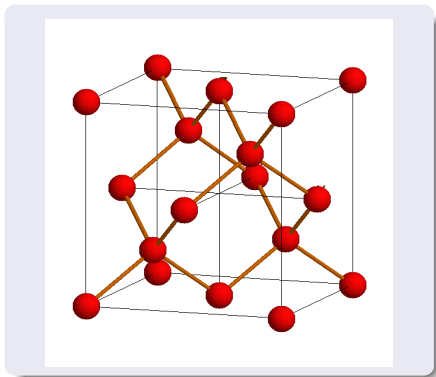
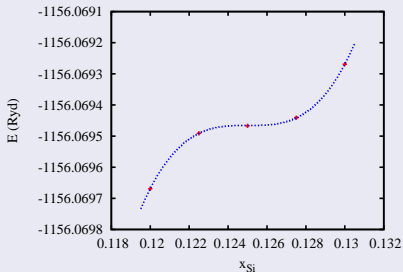
New!

- diamond structure most stable
- pressure induced phase transition to β -tin structure



LTO(Γ)-Phonon in Silicon

ASA+ Code



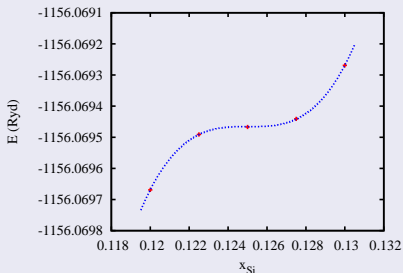
Bad

- no stable Si position # nature

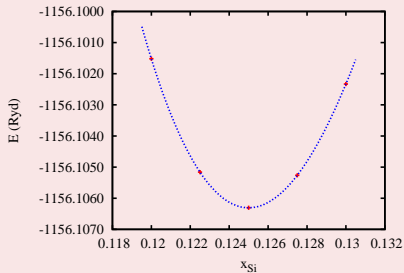


LTO(Γ)-Phonon in Silicon

ASA+ Code



Full-Potential Code



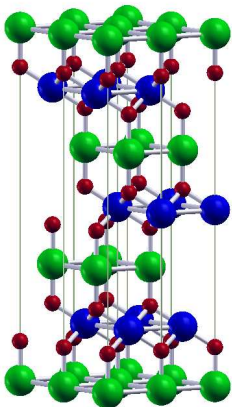
New!

● phonon frequency: $f_{calc} = 15.34$ THz ($f_{exp} = 15.53$ THz)



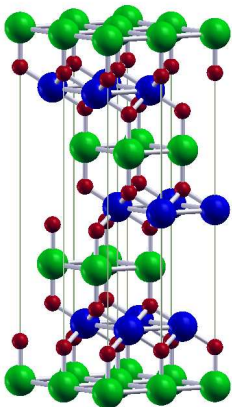
Structure Optimization in PdCoO₂

Delafossite
Structure

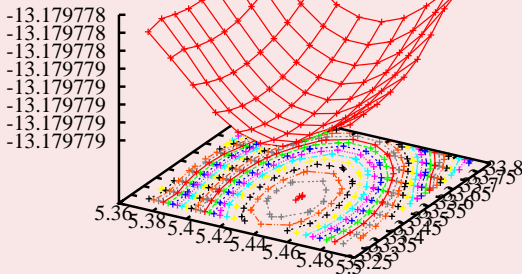


Structure Optimization in PdCoO₂

Delafossite
Structure



Total energy surface



Structure Optimization in PdCoO₂

Structural Data

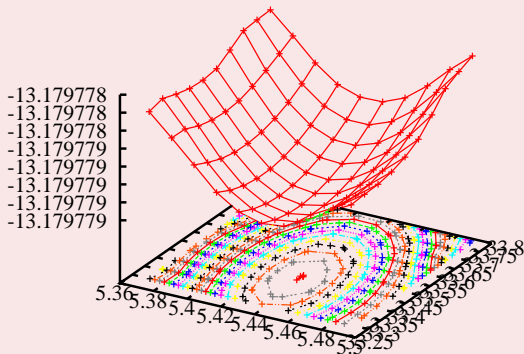
experiment

- $a = 2.83 \text{ \AA}$
- $c = 17.743 \text{ \AA}$
- $z_0 = 0.1112$

theory

- $a = 2.8767 \text{ \AA}$
- $c = 17.7019 \text{ \AA}$
- $z_0 = 0.1100$

Total energy surface



VE, R. Frésard, A. Maignan, Chem. Mat. **20**, 2370 (2008)



Summary

Full-Potential ASW Method

- ASA Geometry used for Basis Functions
- MT Geometry used for Density and Potential
- Accurate Total Energies
- $\mathcal{O}(\text{ASA})$ Speed!
- Optical Properties implemented
- LDA+U-Method implemented

What's Next?

- Forces? Automated Structure Optimization?
- Exact Exchange (EXX)? at $\mathcal{O}(\text{ASA})$ speed?



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Acknowledgments

Stuttgart

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Augsburg

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K.-H. Höck, W. Scherer

Darmstadt

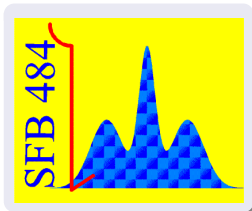
SFB 252

Darmstadt

J. Sticht †

Darmstadt/Jülich

P. C. Schmidt, M. Stephan



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Darmstadt

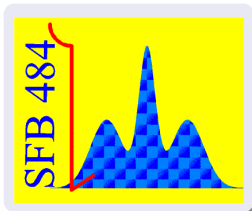
SFB 252

Stuttgart

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Augsburg

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K.-H. Höck, W. Scherer



Darmstadt

J. Sticht †

Darmstadt/Jülich

P. C. Schmidt, M. Stephan

Oran

Thank You for Your Attention!

